Laboratoire Kastler Brossel
Physique quantique et applications
SCIENCES SORBONNE UNIVERSITÉ
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Research School of Physics
ANU College of Science
Australian National University

Engineering and using single mode and multimode squeezed states of light

15/04/2021

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Plan of the talk

- Introduction: Quantum light
- A Source Independent Quantum Random Number Generator (QRNG)
- Generating multimode quantum resources with spectral pump shaping

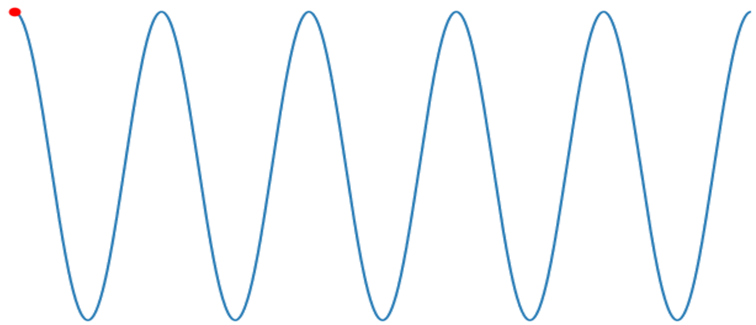
Introduction

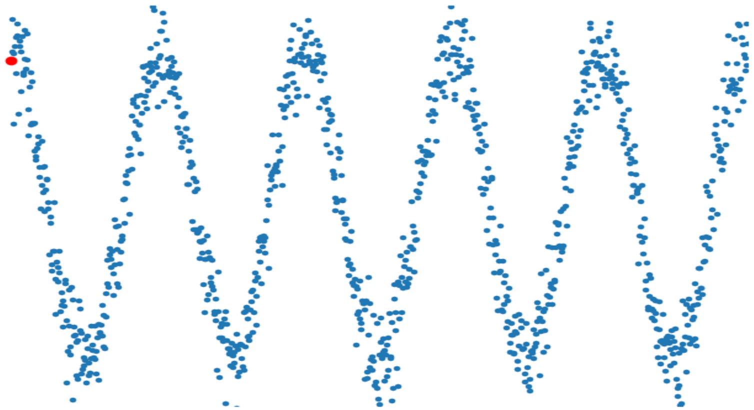
Australian National University

- Classical versus quantum light.
  - Classical

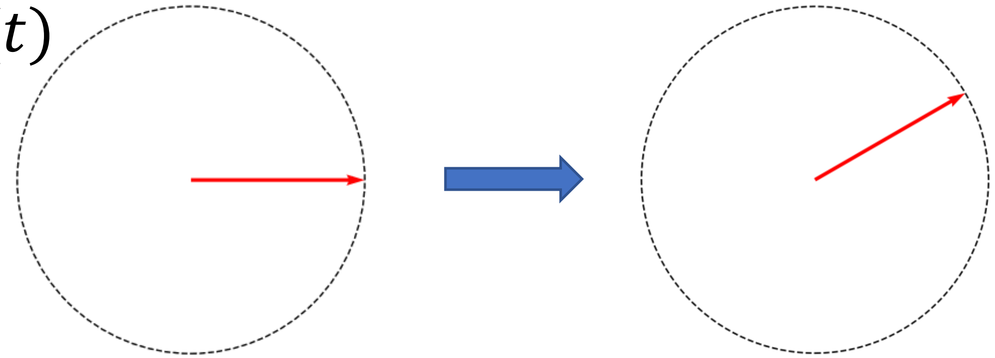
- Quantum

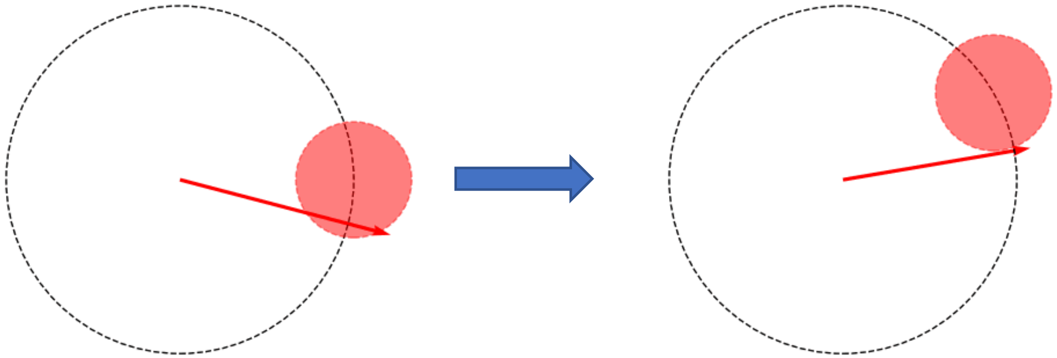
$E(t)$





$\underline{E}\,(t)$





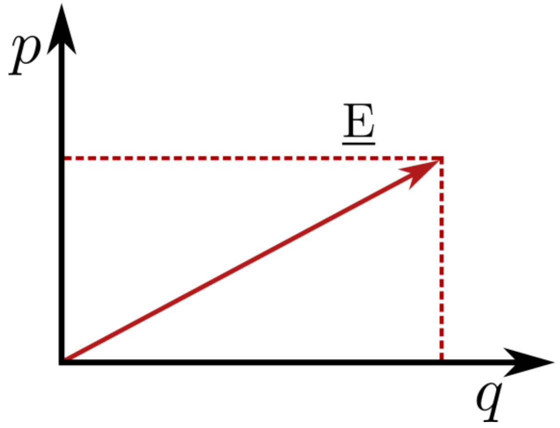
$E(t) = \Re(\underline{E}(t))$

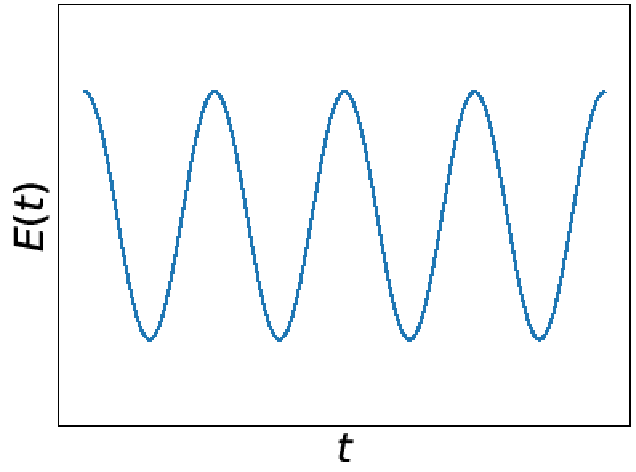
Introduction

- The quadratures

$$E(t) = q\cos(\omega t) + p\sin(\omega t)$$

$$\hat{q} \qquad\qquad \hat{p}$$

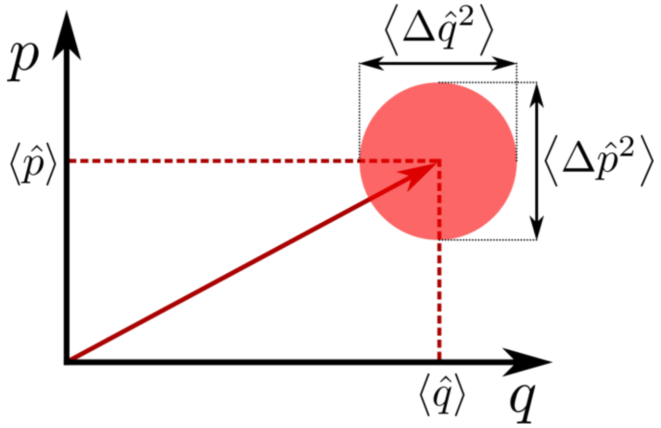


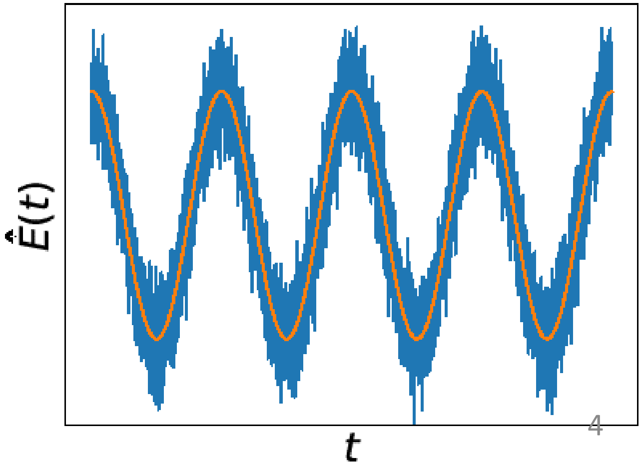


Continuous variables

Heisenberg uncertainty relation:

$$\langle \Delta\hat{q}^2\rangle\langle \Delta\hat{p}^2\rangle \geq cst$$



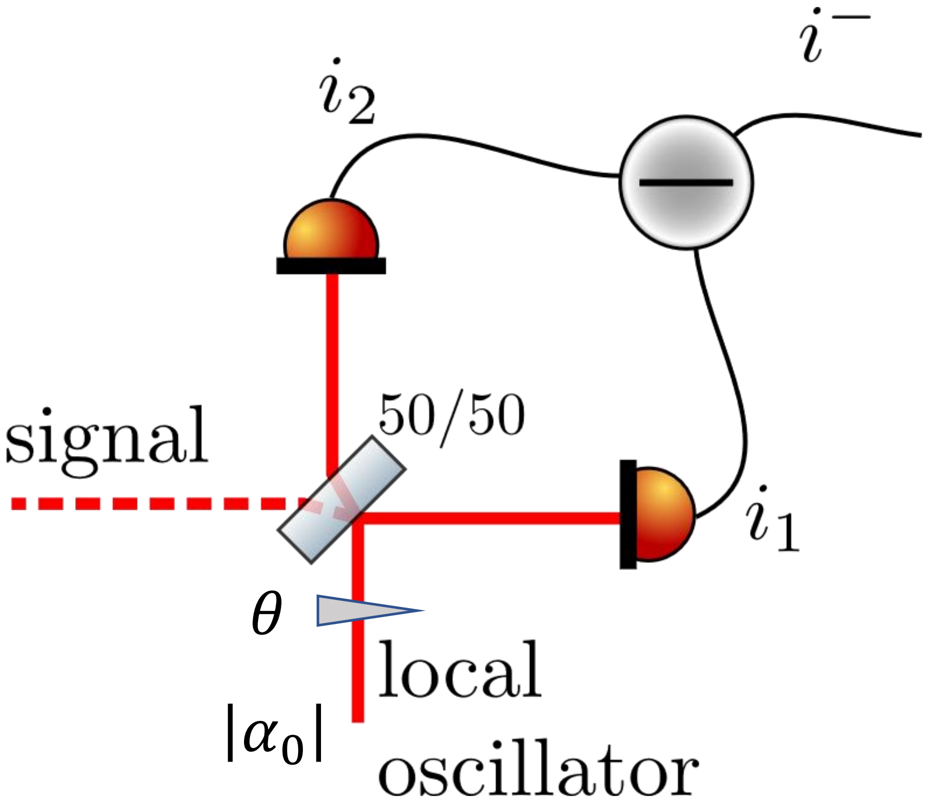


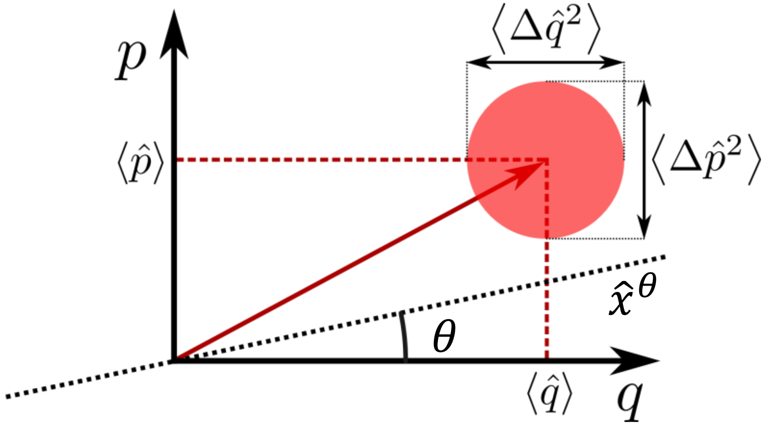
- How do we measure the quadratures in practice:

-> Homodyne detection

$$|\alpha_0|^2 \gg \langle \Delta\hat{q}^2 \rangle$$
$$\langle \Delta\hat{p}^2 \rangle$$

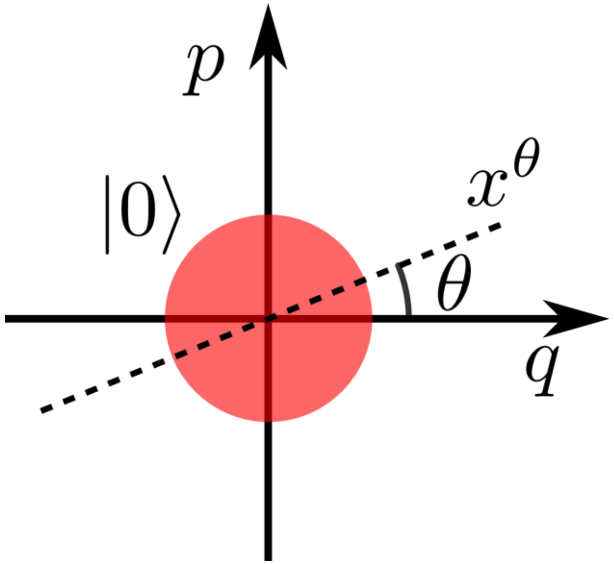
$$\hat{\imath}^- \propto |\alpha_0| \underbrace{(\hat{q}\cos\theta + \hat{p}\sin\theta)}_{\hat{x}^\theta}$$

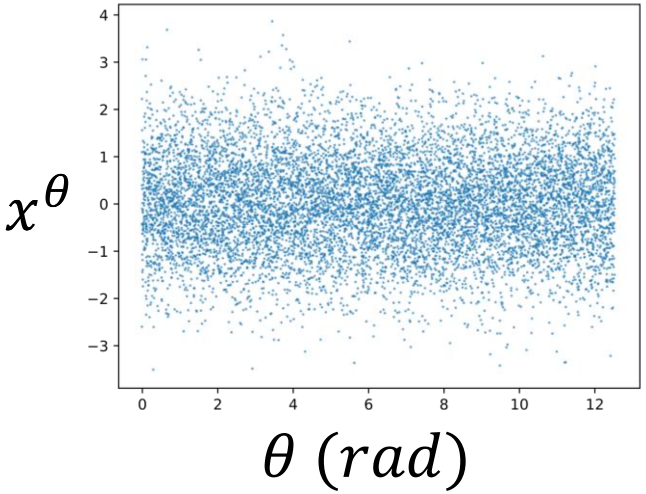




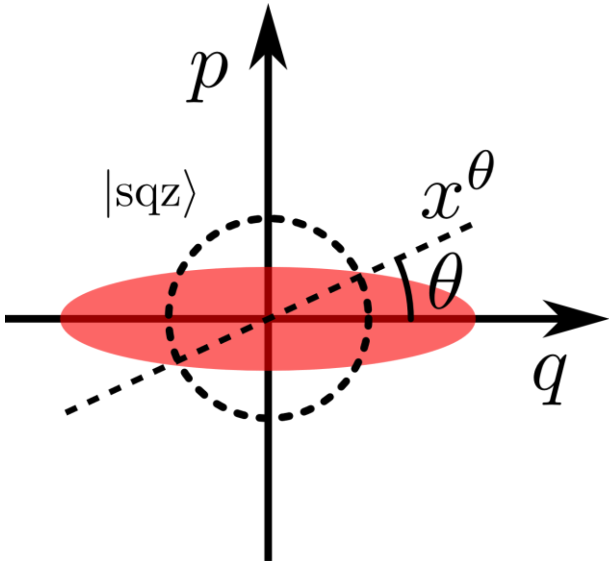
Introduction

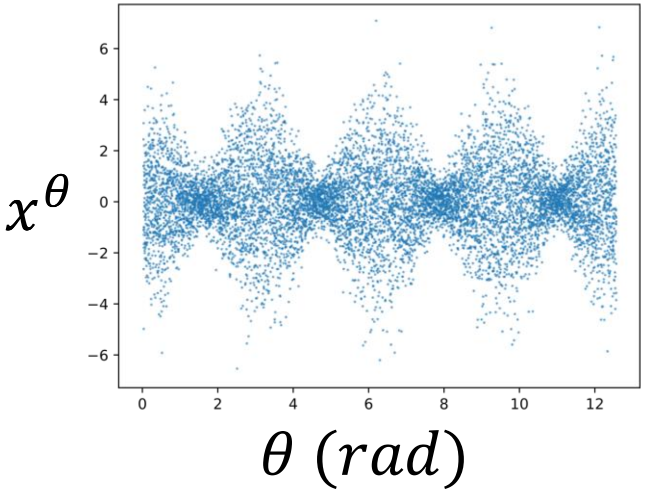
- Vacuum field





- Squeezed vacuum





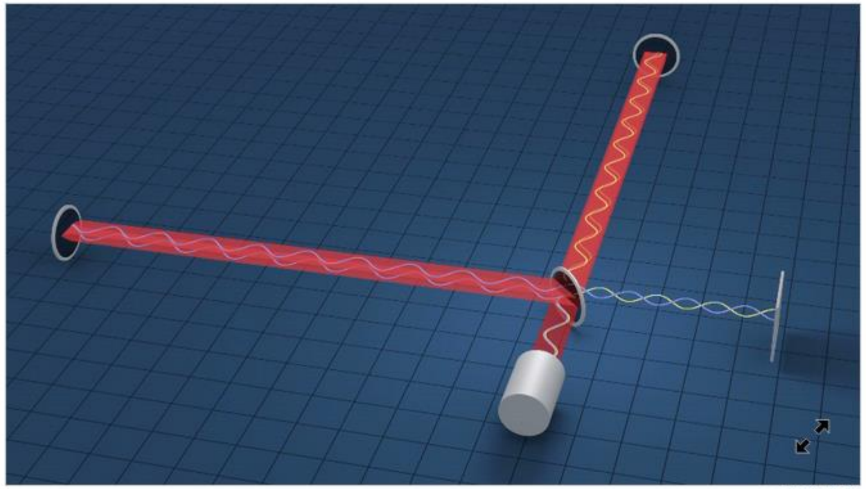
Introduction

- Quadrature squeezed below **shot-noise limit**
- Applications
  - Generation of entangled EPR state for quantum communication
  - Quantum Information Processing
  - Quantum metrology, such as gravitational wave sensing [1, 2]

Squeezing More from Gravitational-Wave Detectors

December 5, 2019 • *Physics* 12, 139

New hardware installed in current gravitational-wave detectors uses quantum effects to boost sensitivity and increase the event detection rate by as much as 50%.



T. Pyle/LIGO

Ball, Philip. "Focus: Squeezing More from Gravitational-Wave Detectors." *Physics* 12 (2019): 139.

[1] Acernese, F., et al. "Increasing the astrophysical reach of the advanced Virgo detector via the application of squeezed vacuum states of light." Physical Review Letters 123.23 (2019): 231108.
[2[Tse, M., et al. "Quantum-enhanced advanced LIGO detectors in the era of gravitational-wave astronomy." Physical Review Letters 123.23 (2019): 231107.

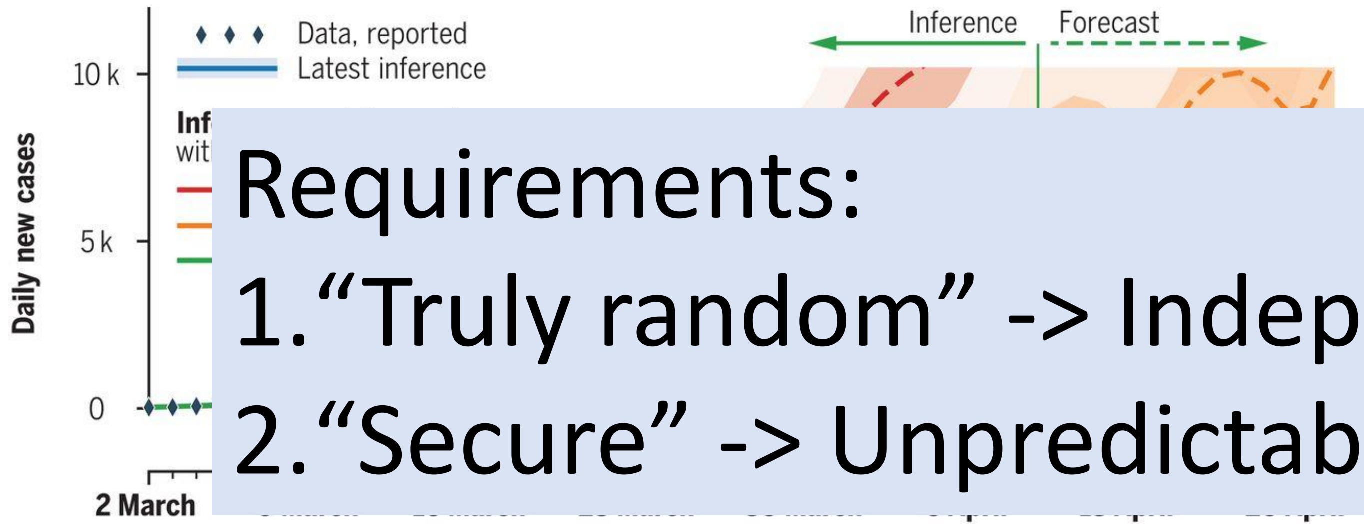
Plan of the talk

- Introduction: Quantum light → Vacuum fluctuations, squeezed states

- A Source Independent Quantum Random Number Generator (QRNG)

- Generating multimode quantum resources with spectral pump shaping

- What are random number useful for ?
  - Computer simulations
  - Encryption, secure communications



Jonas Dehning et al. Science 2020;369:eabb9789

Requirements:
1. "Truly random" -> Independent, Uniform
2. "Secure" -> Unpredictable

- How to produce random numbers ?

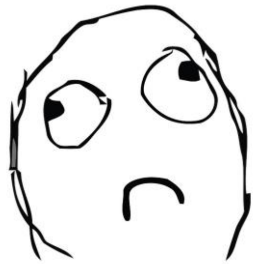
-> Algorithmically

- Digits of $\pi = 3.14159265358979323846264338327 95$
- $d_n = (4 \times d_{n-1} + 1)\ \%\ 9$ 34867201534867201534867201534867 2

} Pseudo Random Number Generators

Fast | **No hardware cost** | **Reproducible** | **Deterministic => ~~secure~~**



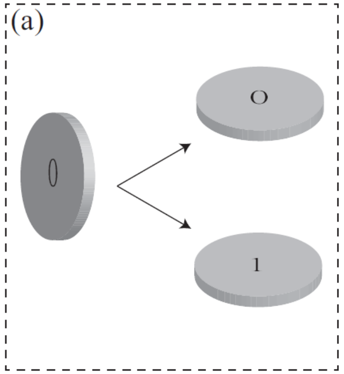



- How to produce random numbers ?

-> From a “naturally random” phenomena:

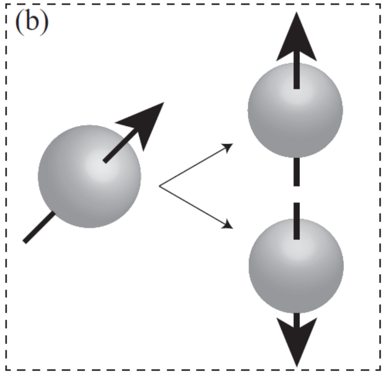
- Classical RNG

→ Predictable in principle, security relies on the trust in the model.

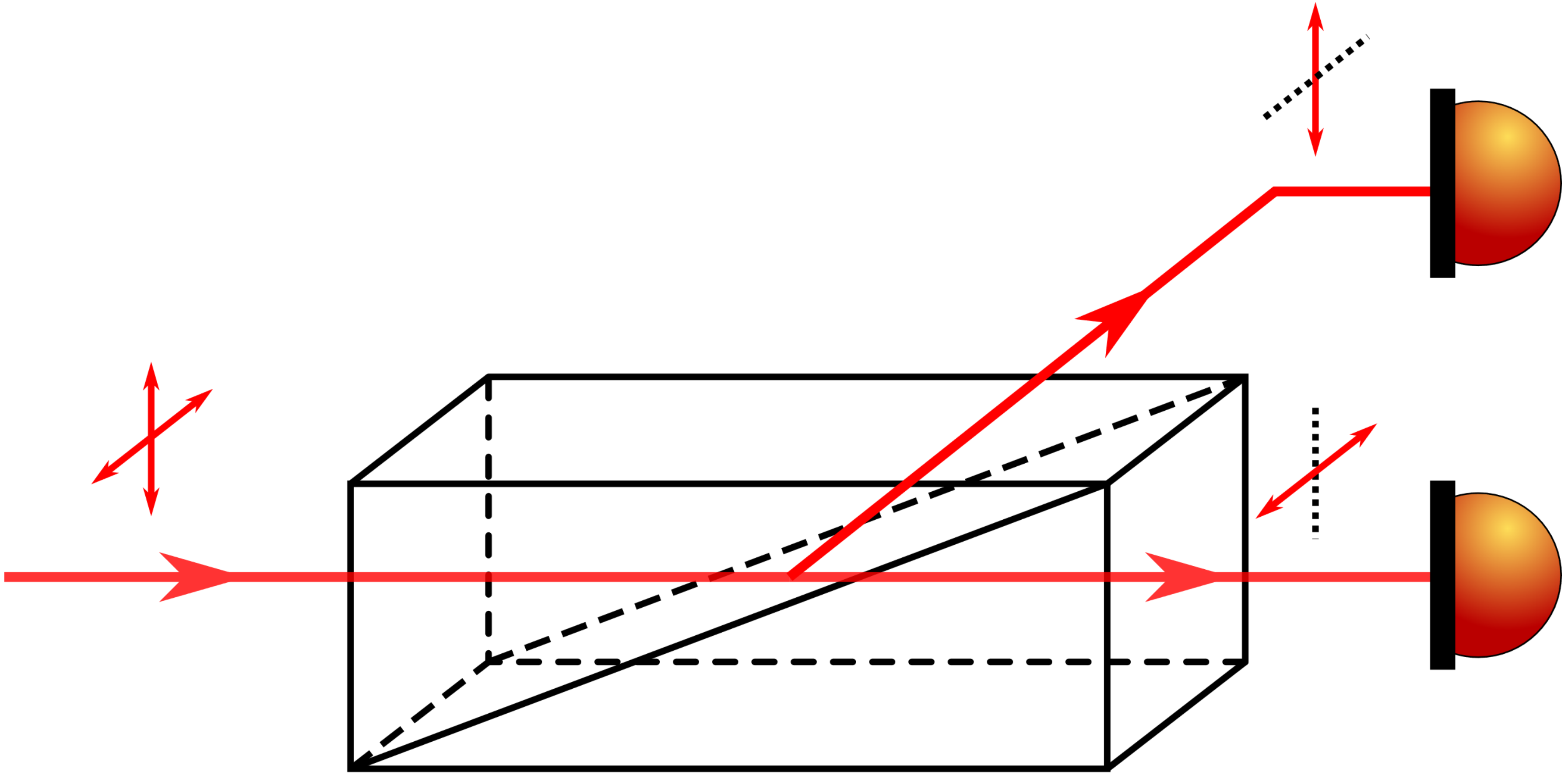


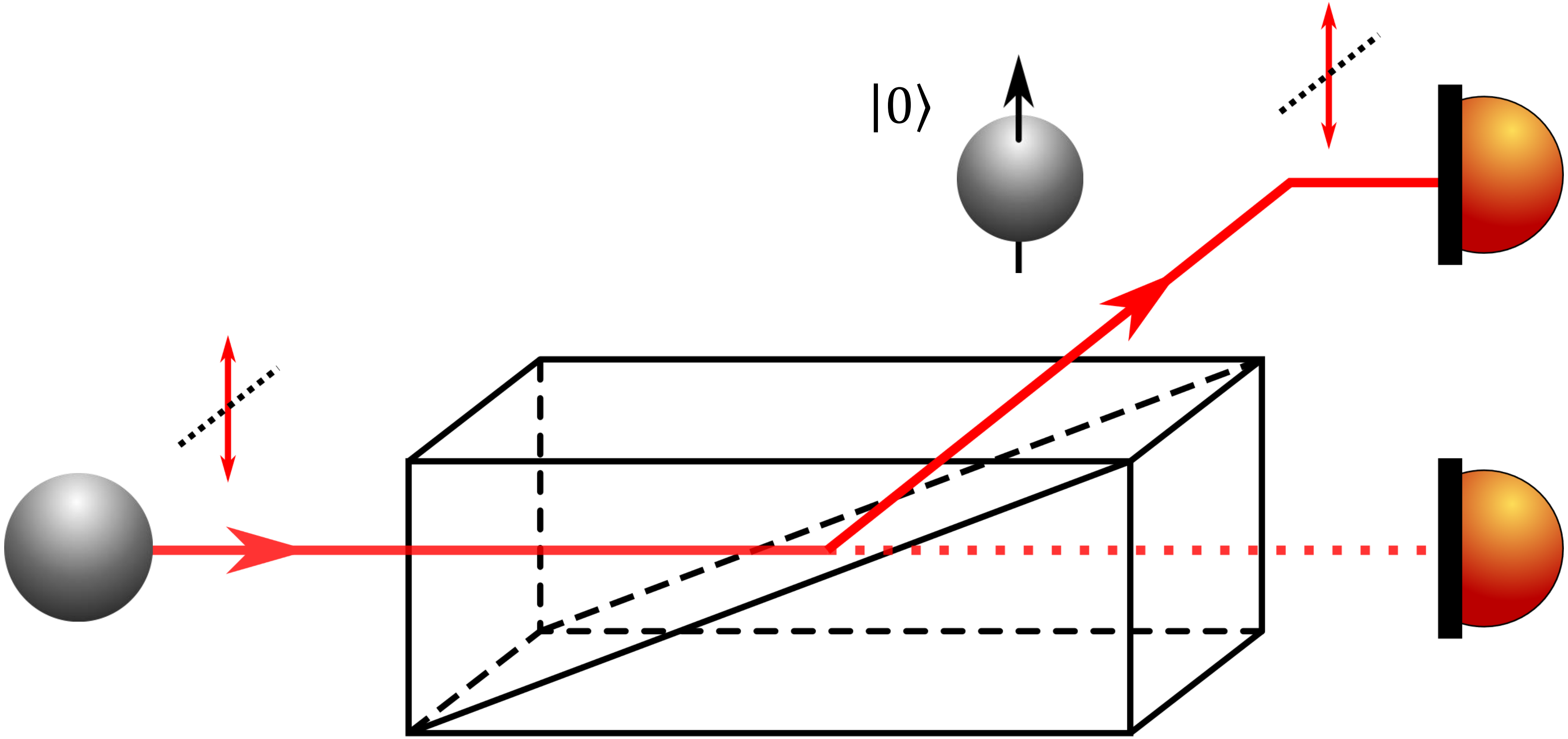
- Quantum RNG:

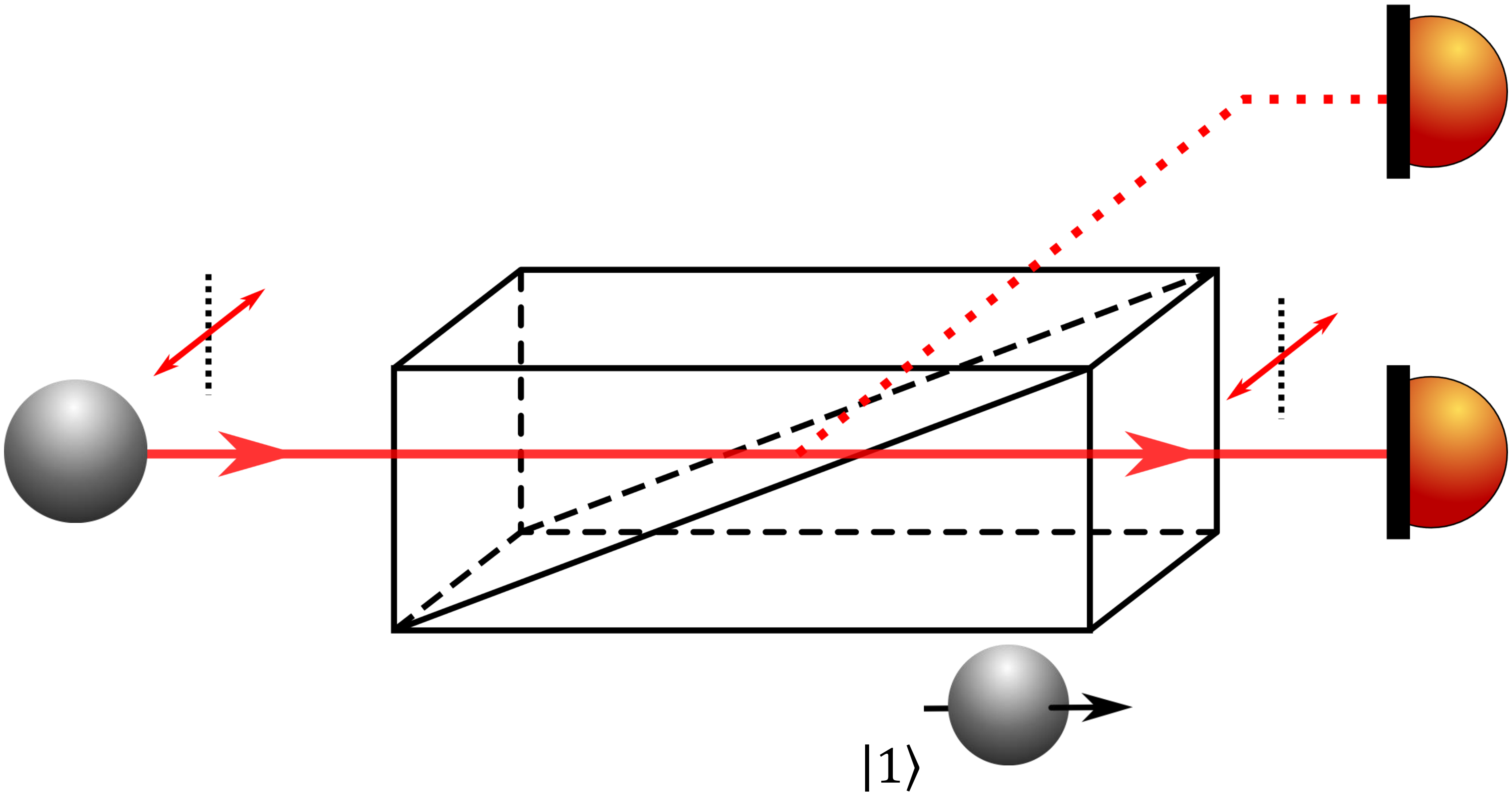
→ Fundamental unpredictability Born’s rule



→ Statistically perfect source of 0s and 1s.

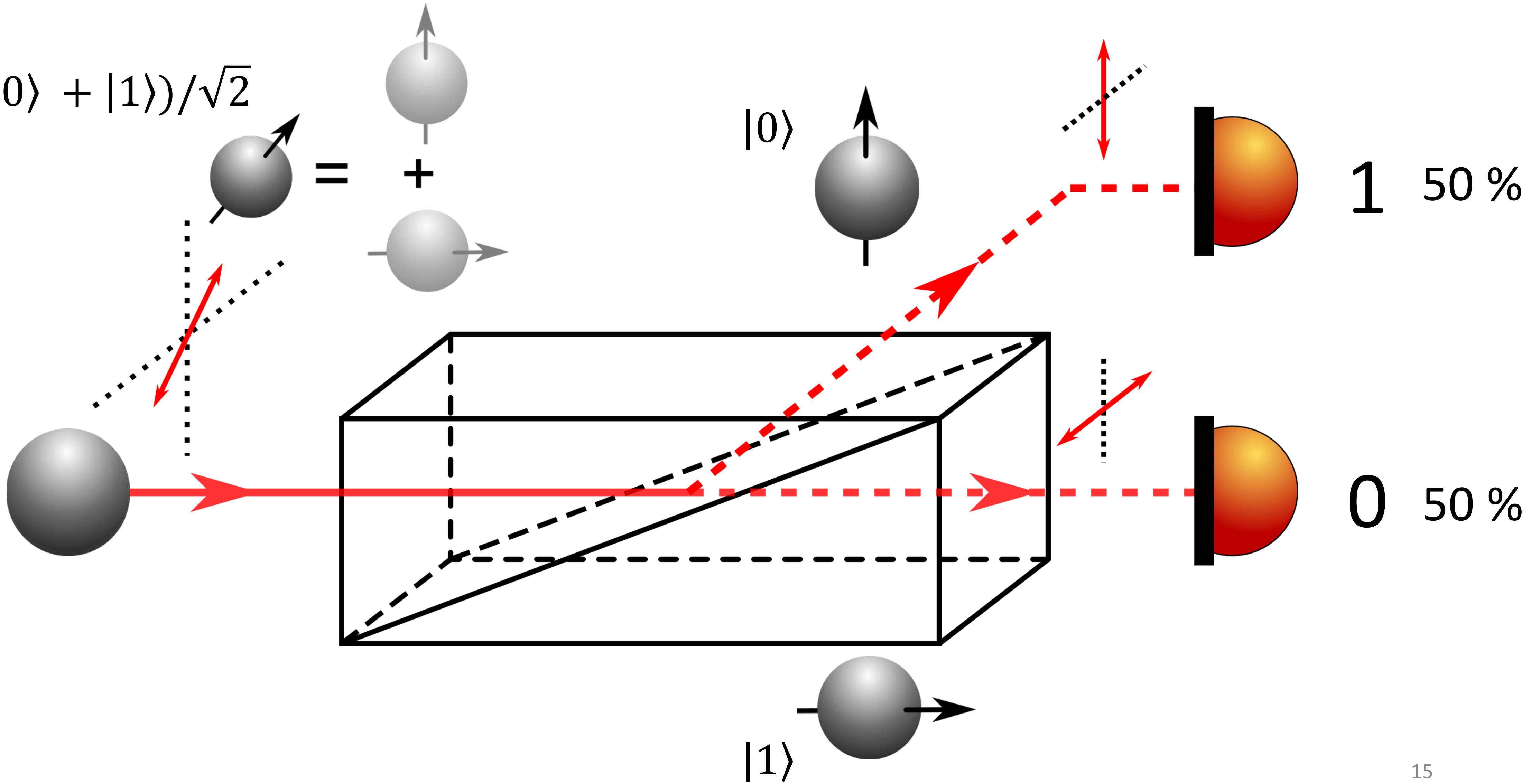



|0⟩

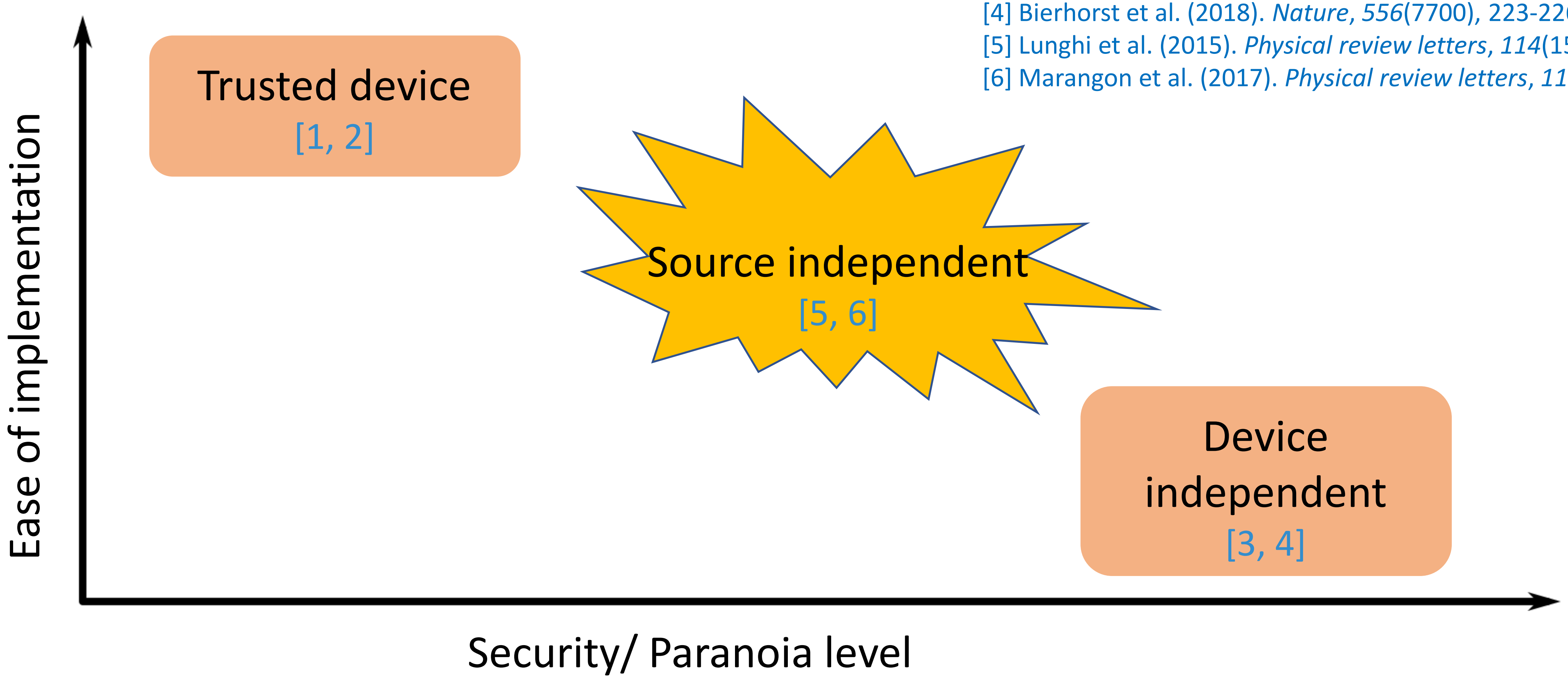


|1⟩

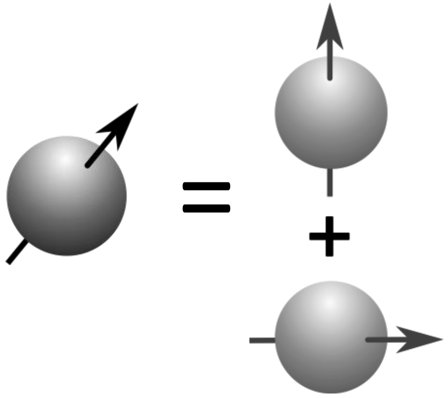
$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$

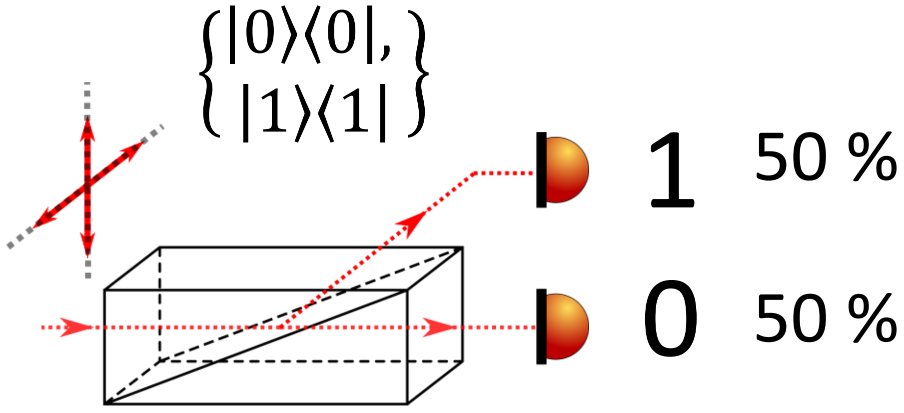


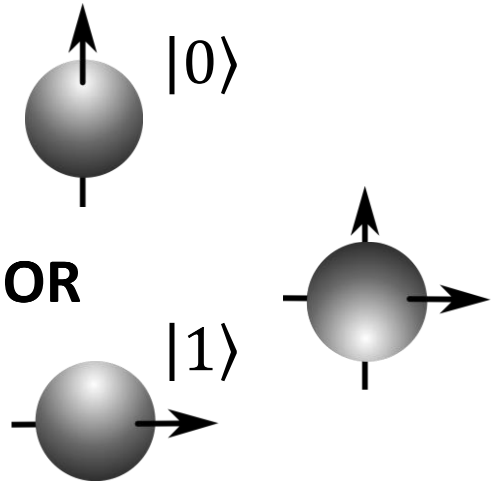
[1] Symul et al. (2011). *Applied Physics Letters*, *98*(23), 231103.
[2] Abellán et al. (2019). *Optics express* 22.2: 1645-1654
[3] Liu et al. (2018). *Physical review letters*, *120*(1), 010503.
[4] Bierhorst et al. (2018). *Nature*, *556*(7700), 223-226.
[5] Lunghi et al. (2015). *Physical review letters*, *114*(15), 150501.
[6] Marangon et al. (2017). *Physical review letters*, *118*(6), 060503.

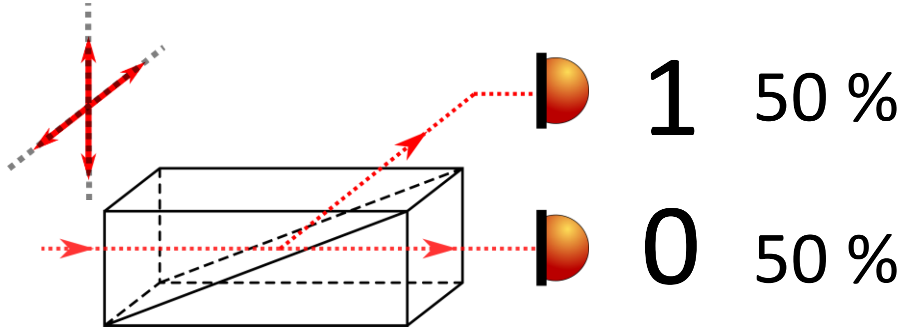


$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$



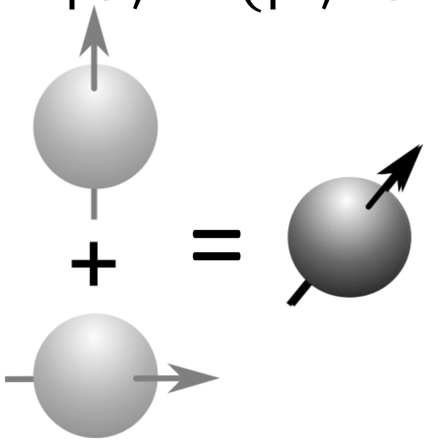


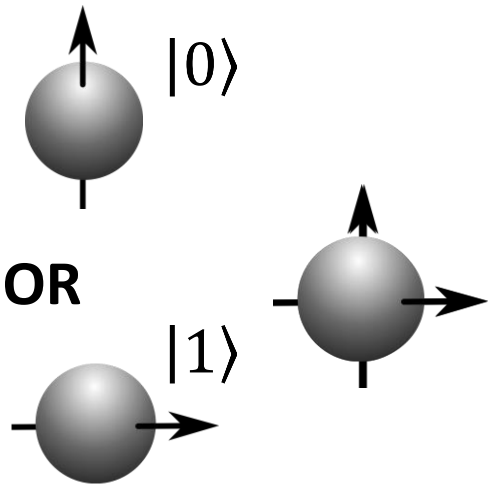




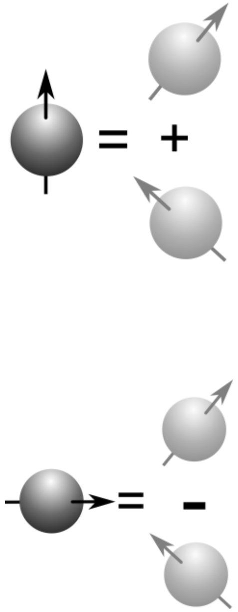
$$\rho = (|0\rangle\langle 0| + |1\rangle\langle 1|)/2$$

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

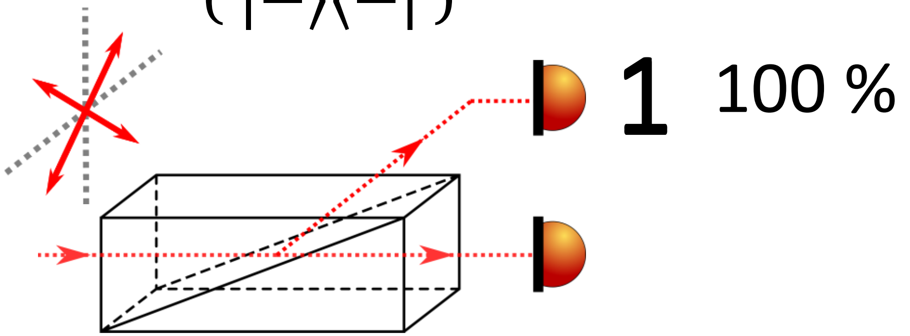


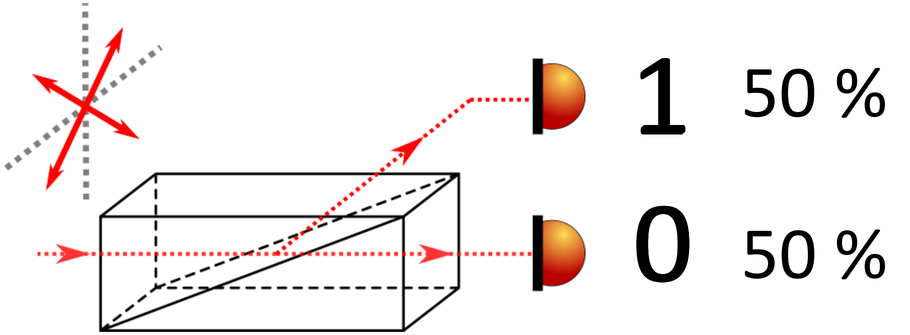


$$\rho = (|0\rangle\langle 0| + |1\rangle\langle 1|)/2$$



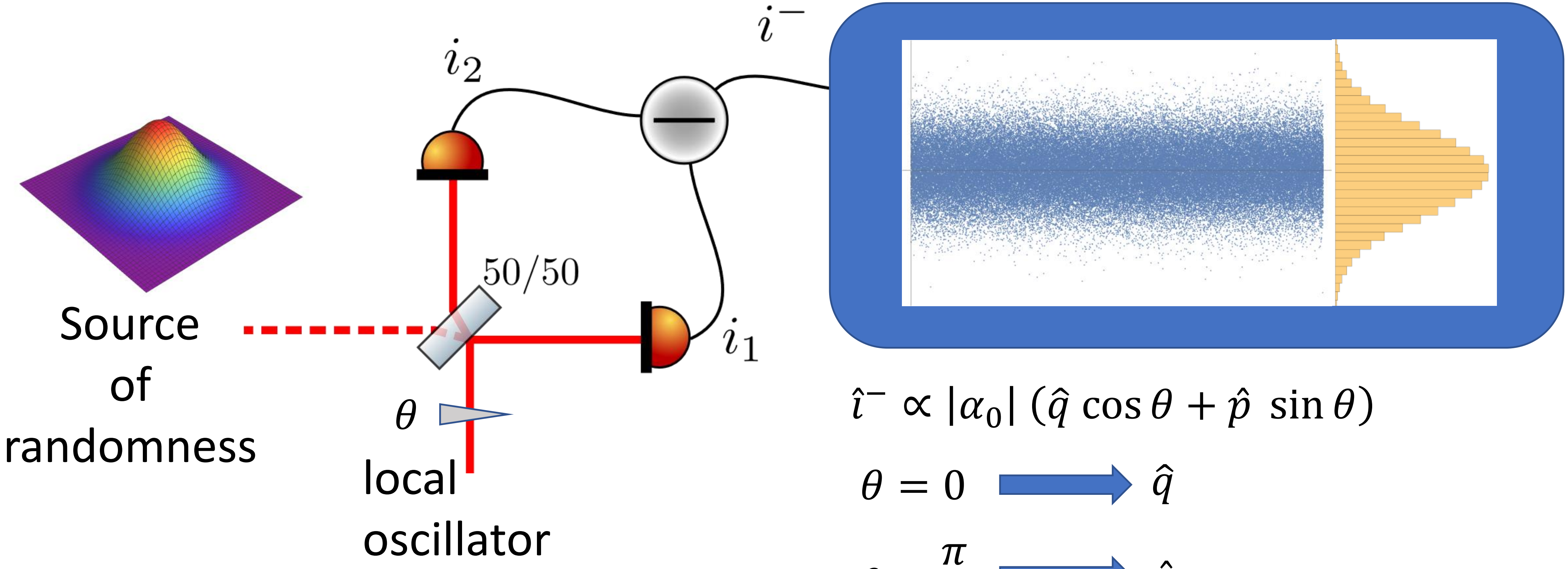
$$\left\{ \begin{matrix} |+\rangle\langle +|, \\ |-\rangle\langle -| \end{matrix} \right\}$$





- Discrete Variable QRNG
- Continuous variable QRNG

$\hat{Q}_{\delta q}, \hat{P}_{\delta p}$

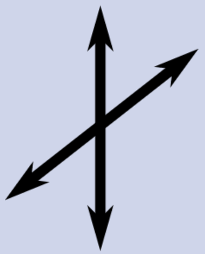
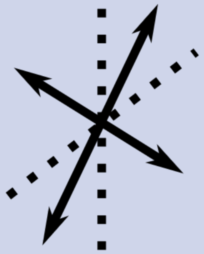
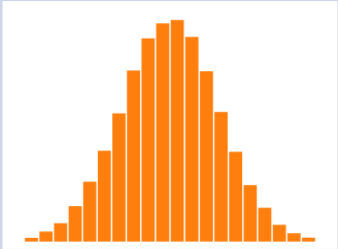
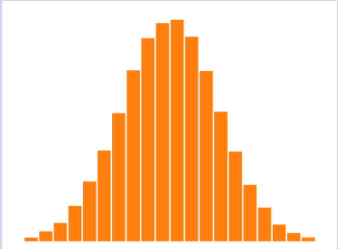
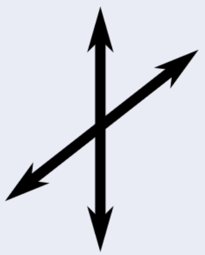
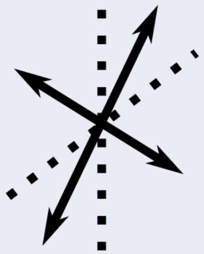
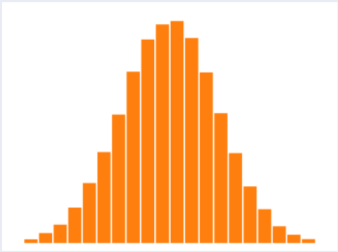


$$\hat{\imath}^- \propto |\alpha_0| \left(\hat{q}\cos\theta + \hat{p}\ \sin\theta\right)$$

$\theta = 0 \Rightarrow \hat{q}$

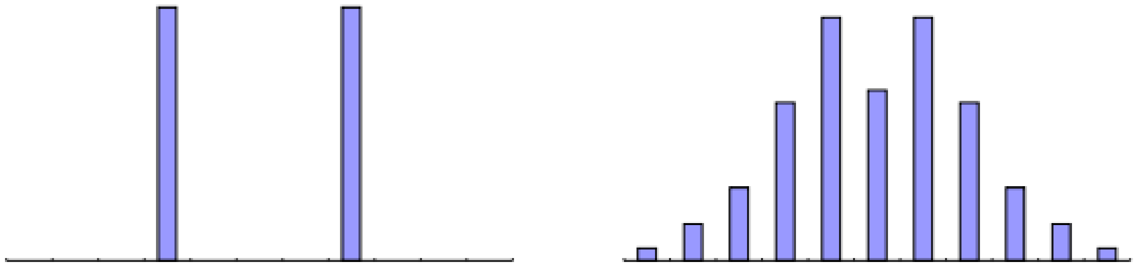
$\theta = \frac{\pi}{2} \Rightarrow \hat{p}$

- Source independent QRNG

	Discrete Variables		Continuous Variables	
"Unsafe" mixed source	 50%/50%	 50%/50%	$\hat{Q}_{\delta q}$ 	$\hat{P}_{\delta p}$ ("check") 
"Safe" Pure quantum source	 50%/50%	 100%/0%	$\hat{Q}_{\delta q}$ 	$\hat{P}_{\delta p}$("check") 

- How to quantify the randomness amount ?

  - Variance is not enough



(a) low entropy distribution (b) high entropy distribution

  - Relevant quantity is the Min-entropy:

$$H_{min}(Q) = -\log_2\left(\max_k\{proba\ (q_k)\}\right)$$

  - When source is untrusted; conditional min-entropy [1]:

$$H_{min}(Q|E) = -\log_2\left(proba^E_{guess}\right)$$

$$H_{min}(Q) \geq H_{min}(Q|E)$$

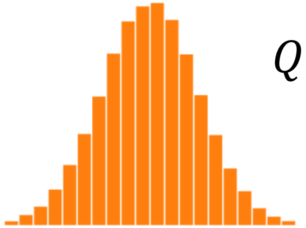
[1] R. Konig, R. Renner, and C. Schaffner, The operational meaning of min- and max-entropy, IEEE Trans. Inf. Theory **55**, 4337 (2009).

- The entropic uncertainty principle:

$$H_{min}(Q|E) + H_{max}(P) \geq -\log_2 c(\delta q, \delta p) \qquad H_{max}(P) = 2\ \log_2 \sum_k \sqrt{proba(p_k)}$$

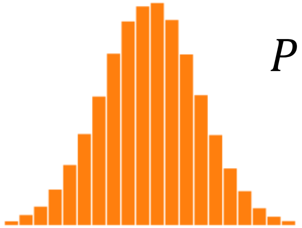
$$\Rightarrow H_{min}(Q|E) \geq H_{low}(P) \stackrel{\text{def}}{=} -H_{max}(P) - \log_2 c(\delta q, \delta p)$$

- “unsafe” state:



Q

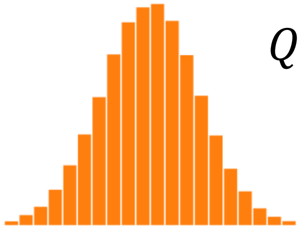
- $H_{min}(Q)$



P (check)

- Low $H_{min}(Q|E)$

- “safe” state:

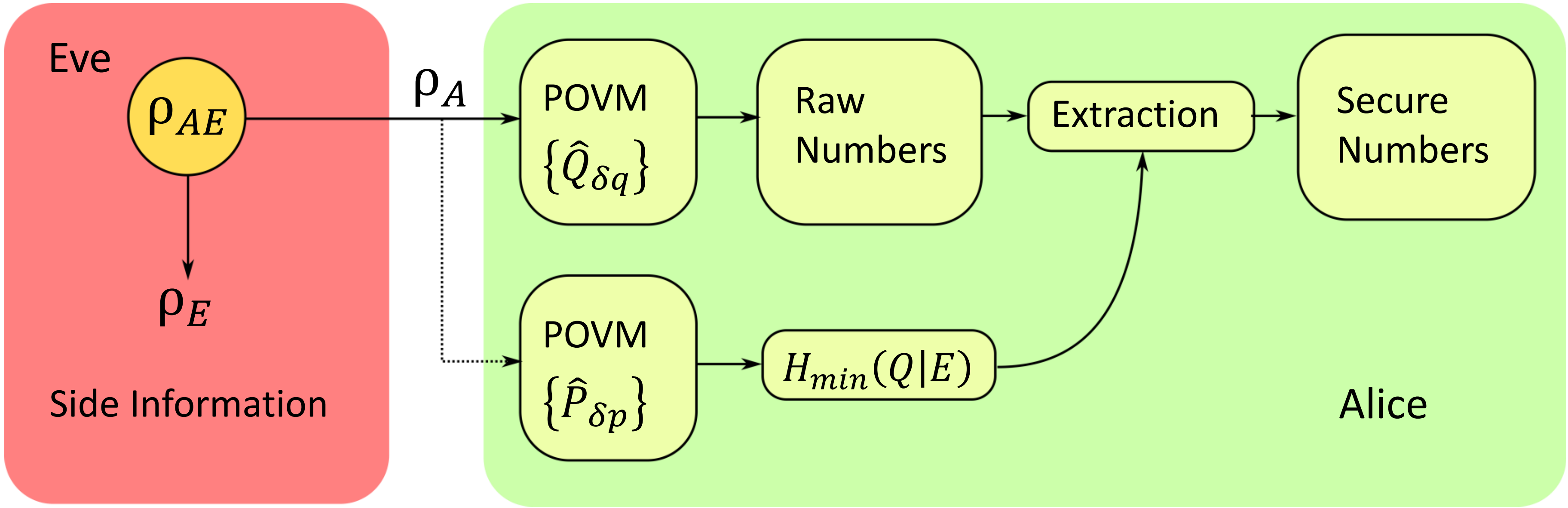


Q

- same $H_{min}(Q)$

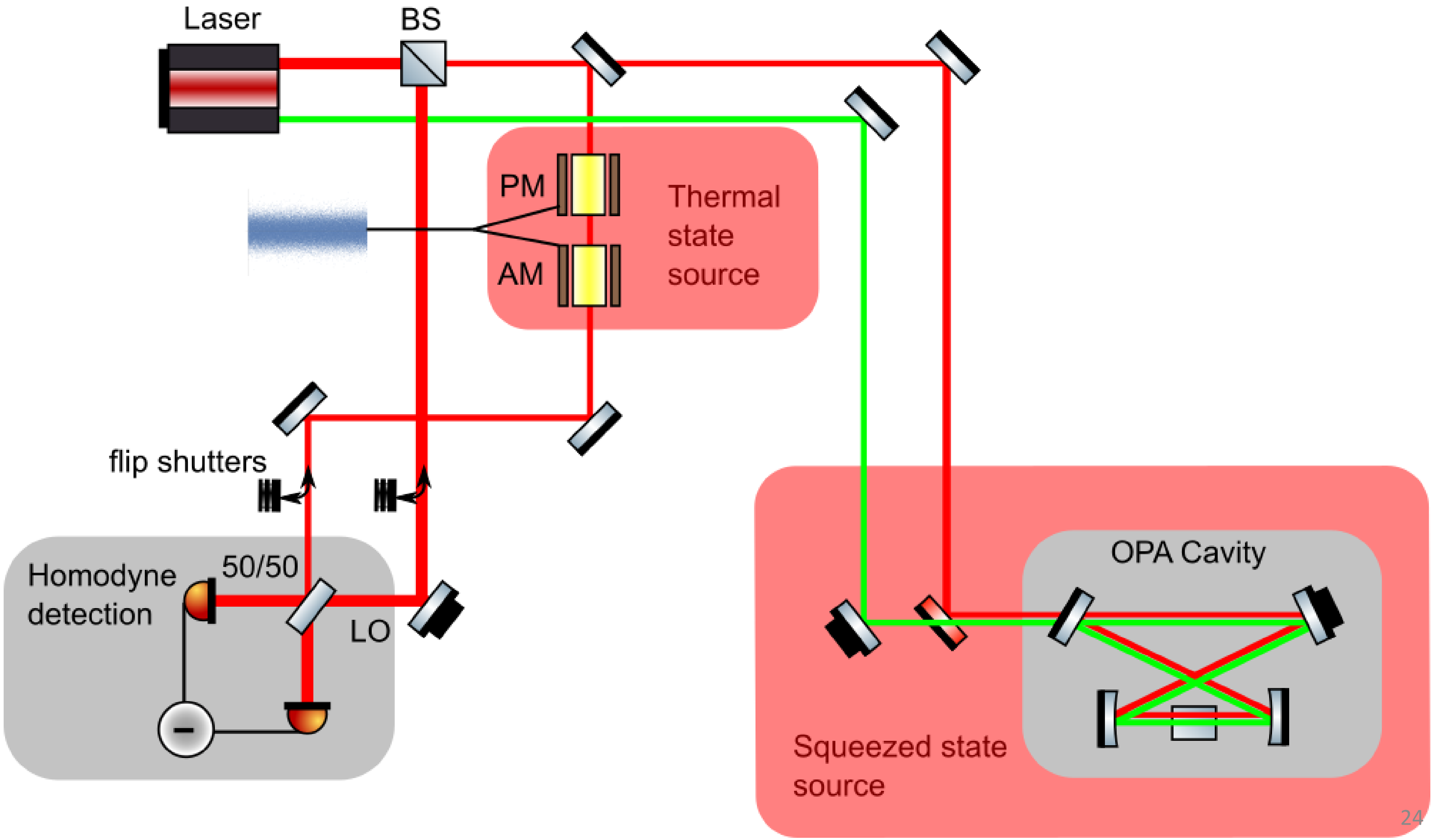
P (check)

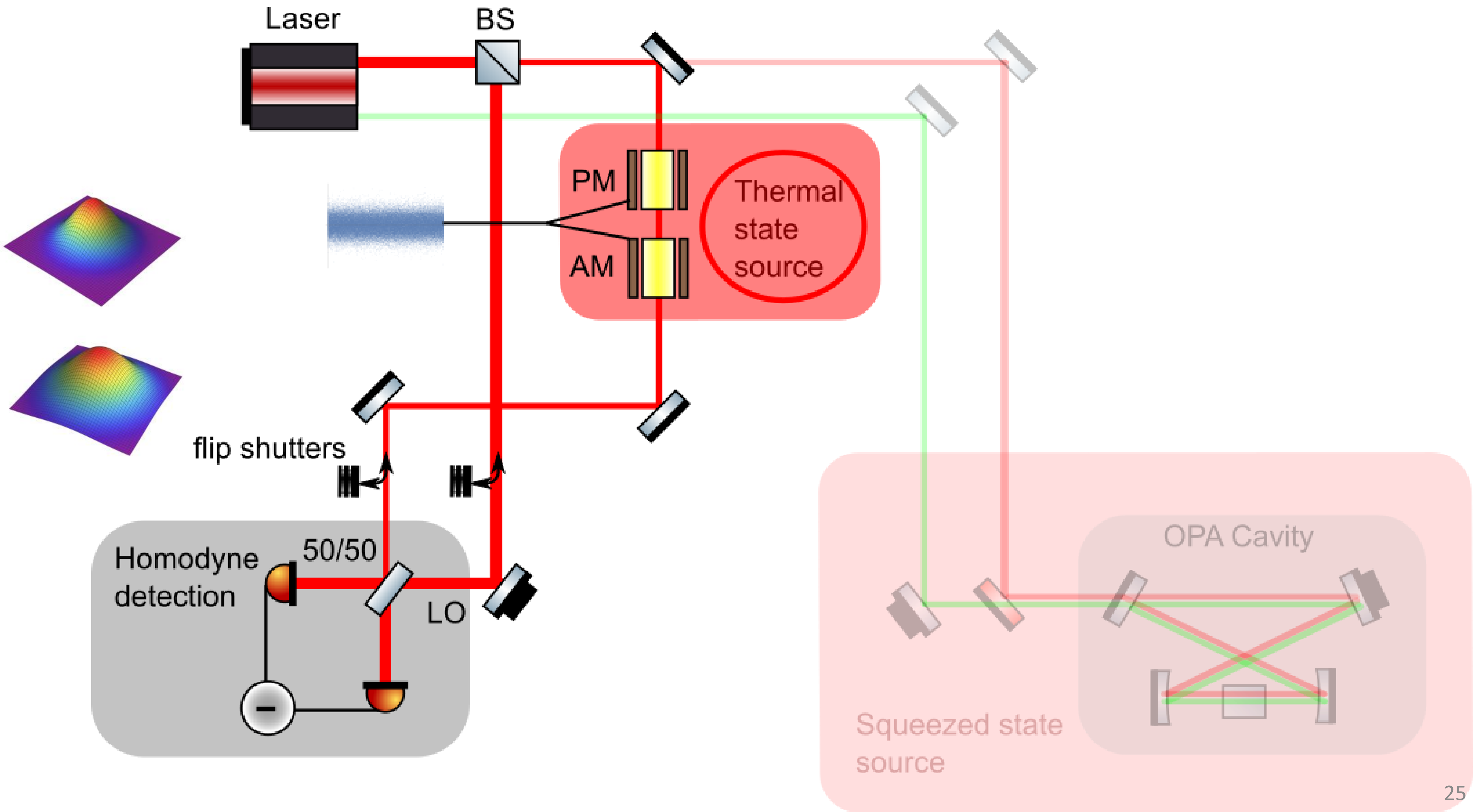
- Higher $H_{min}(Q|E)$



Assumptions:

- Trusted measurement device (untrusted classical noise)
- I.I.D. and bounded source (untrusted)


Laser
BS
PM
AM
Thermal state source
flip shutters
Homodyne detection
50/50
LO
OPA Cavity
Squeezed state source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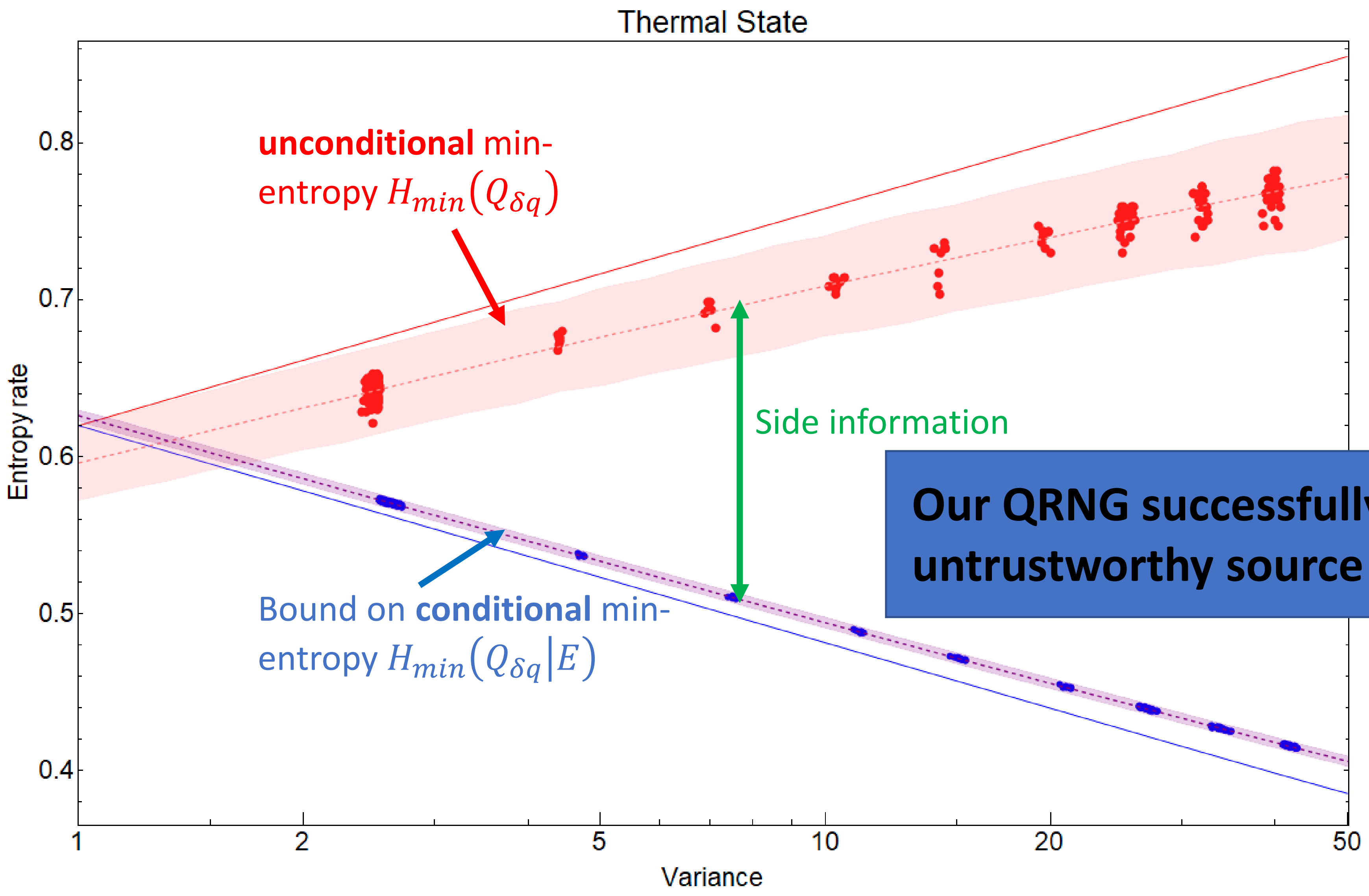
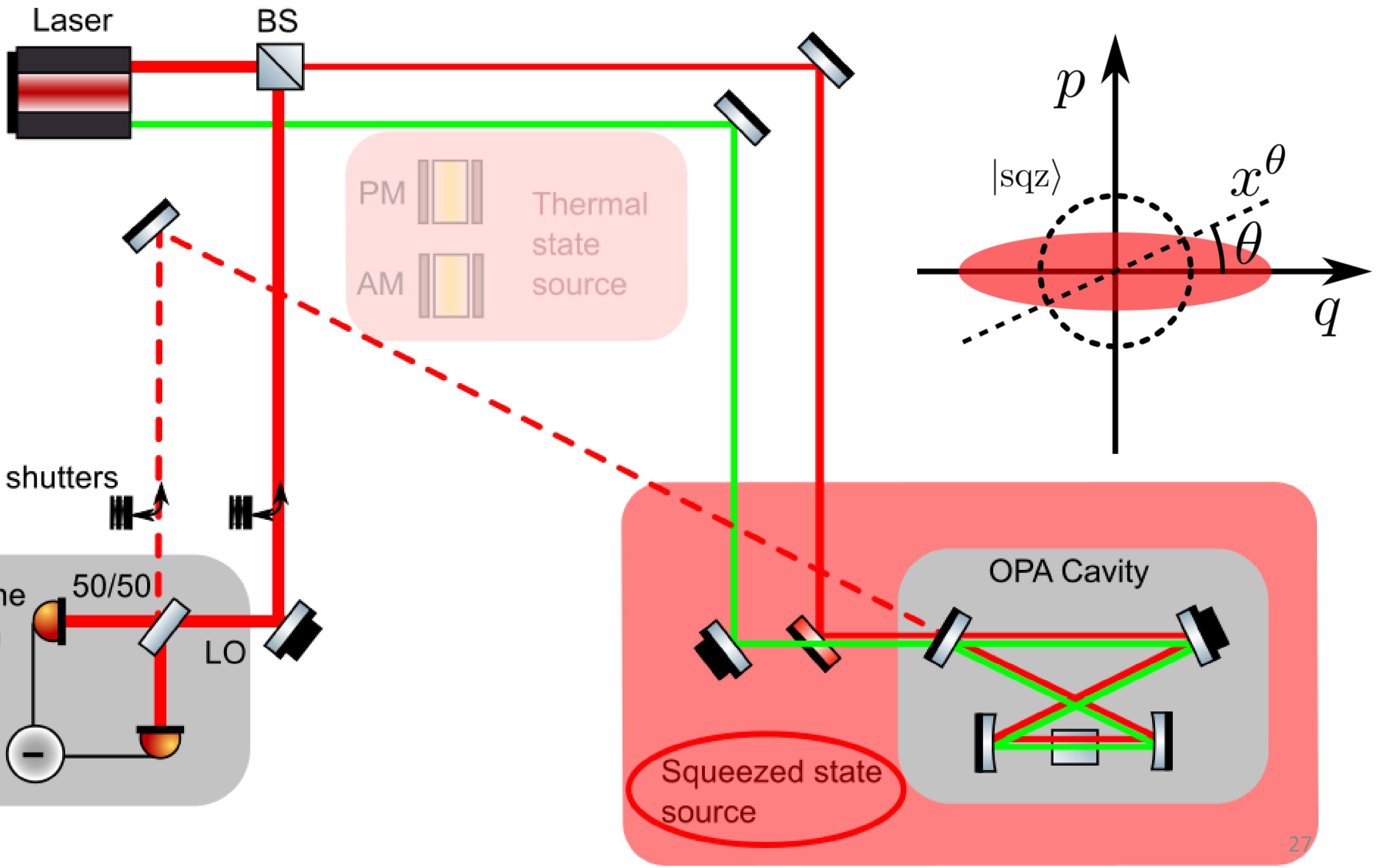
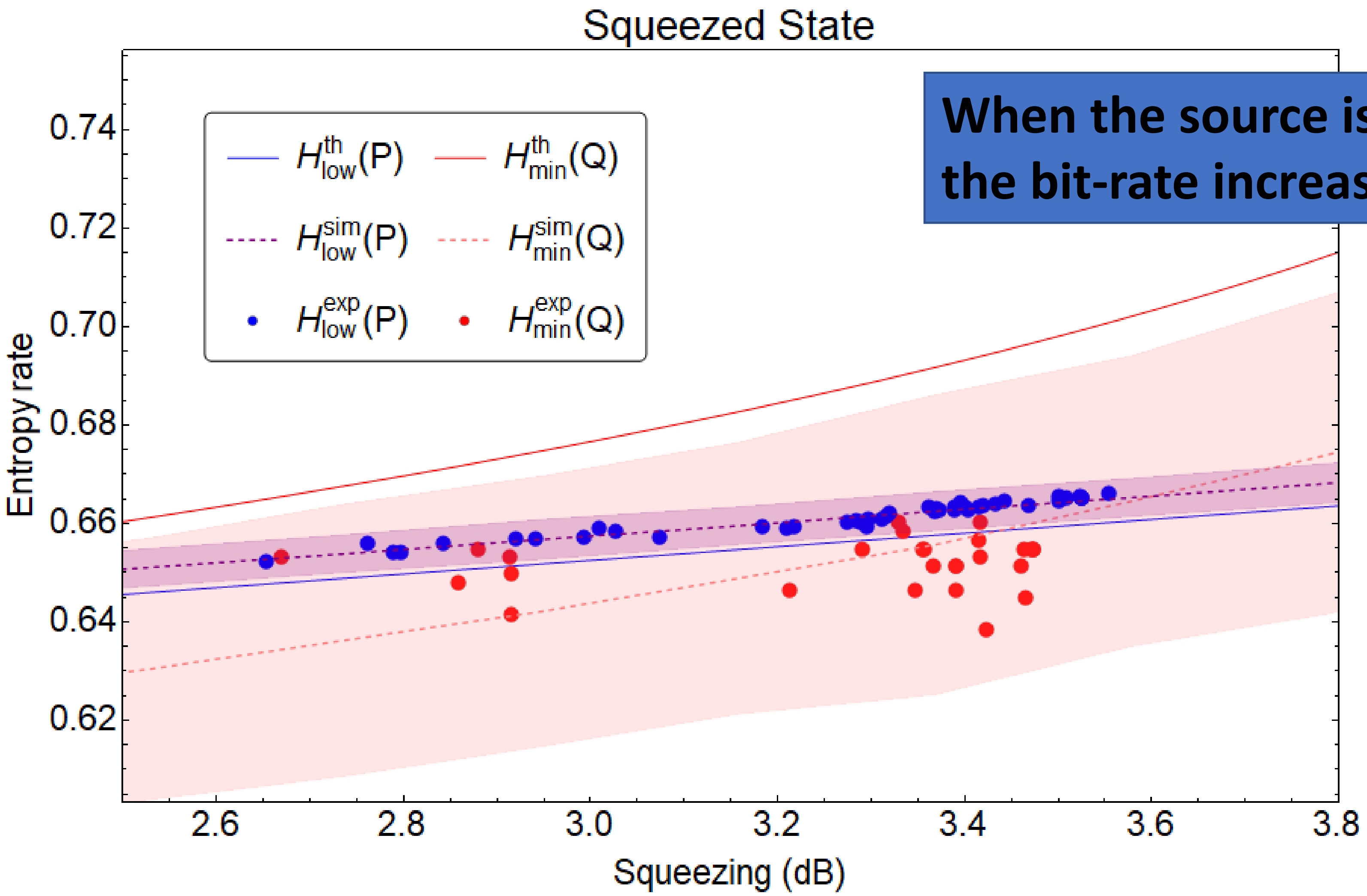


Our QRNG successfully detects an untrustworthy source



Laser
BS
PM
AM
Thermal state source
flip shutters
Homodyne detection
50/50
LO
OPA Cavity
Squeezed state source
p
|sqz⟩
x^θ
θ
q



When the source is trustworthy, the bit-rate increases

- Conclusion
  - We demonstrated a **real-time**, **self-testing** QRNG based on **non-classical light**. **First QRNG** based on **squeezed light**.
  - We tested the QRNG with **different sources** to validate the source independent protocol
  - Typical rate ~10kb/s
- Perspective
  - From proof of principle to high speed QRNG
  - Use squeezed source for other device independent protocol

Real-Time Source-Independent Quantum Random-Number Generator with Squeezed States
Thibault Michel, J.Y. Haw, D. G. Marangon, O. Thearle, G. Vallone, P. Villoresi , P. K. Lam, S.M. Assad

PhysRevApplied.12.034017

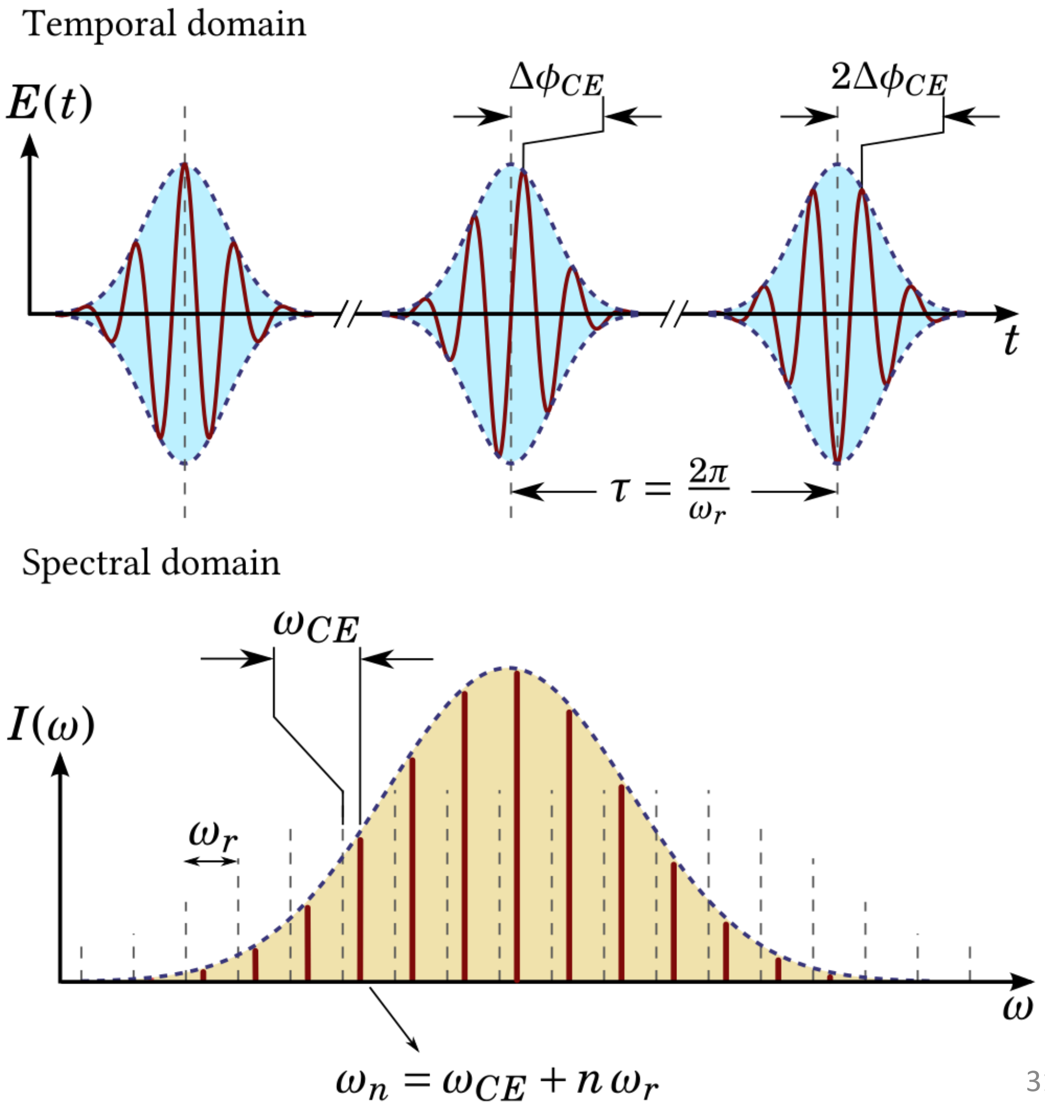
Plan of the talk

- Introduction: Quantum light
  - Vacuum fluctuations, squeezed states
  - Notion of mode, entanglement
- A Quantum Random Number Generator (QRNG)
- Generating multimode quantum resources with spectral pump shaping

Introduction

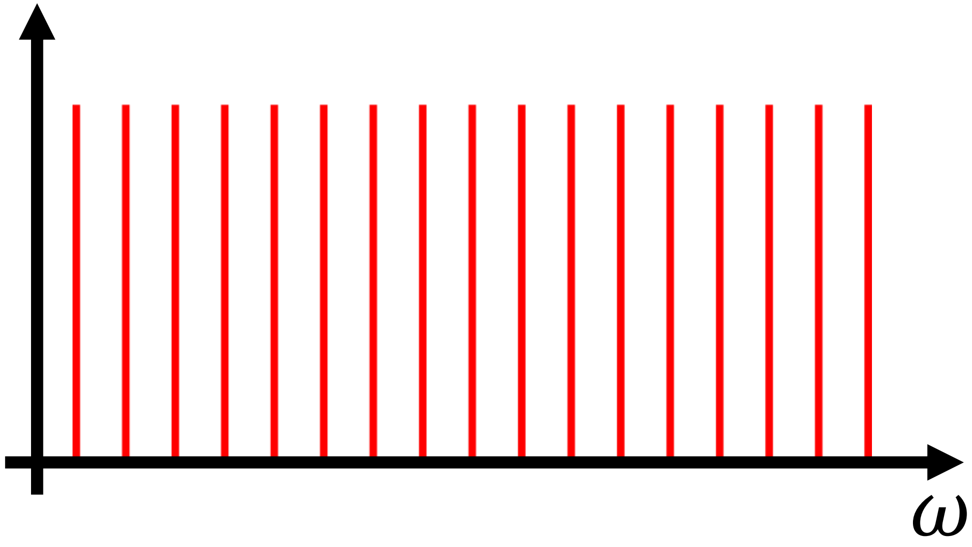
- Femtosecond laser:
  - 100 fs pulses
  - 76 MHz repetition rate
  - High peak intensity ~MW

$$\hat{E}(t) = \sum_k \hat{q}_k \cos(\omega_k t) + \hat{p}_k \sin(\omega_k t)$$

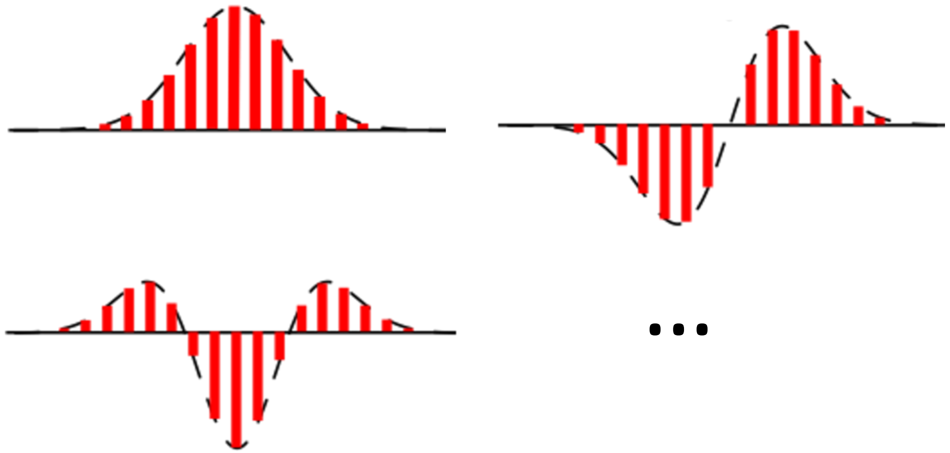


Introduction

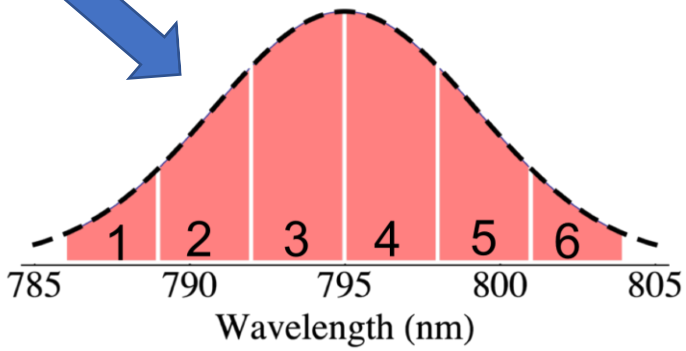
- Mode basis change



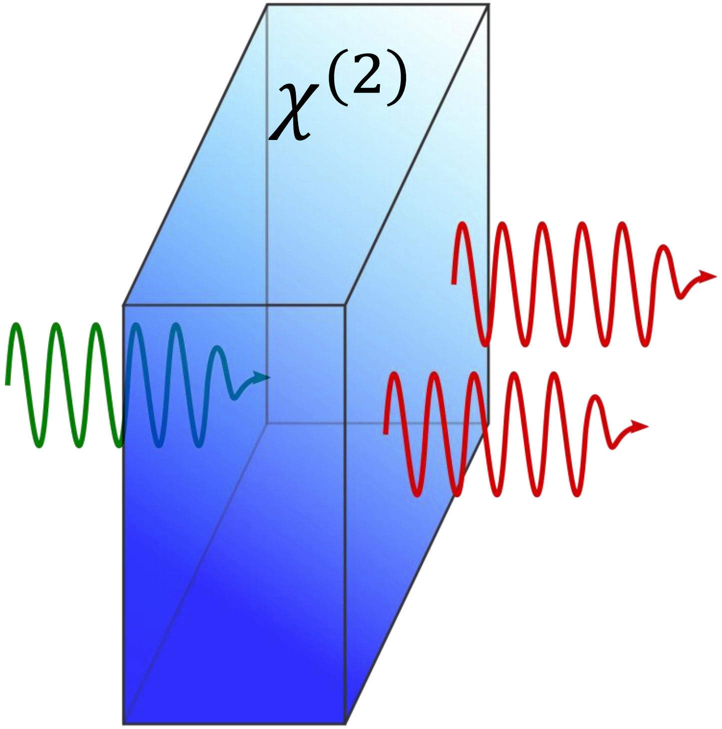
- HG mode basis



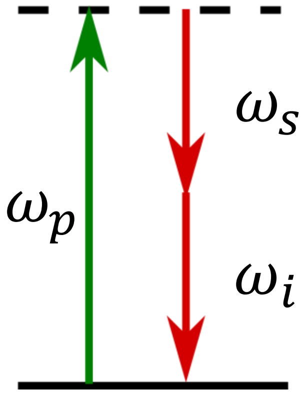
- Frexel basis



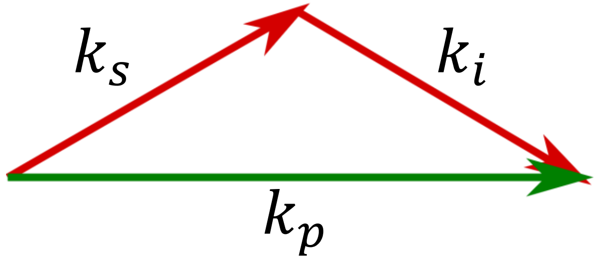
Pump shaping optimization

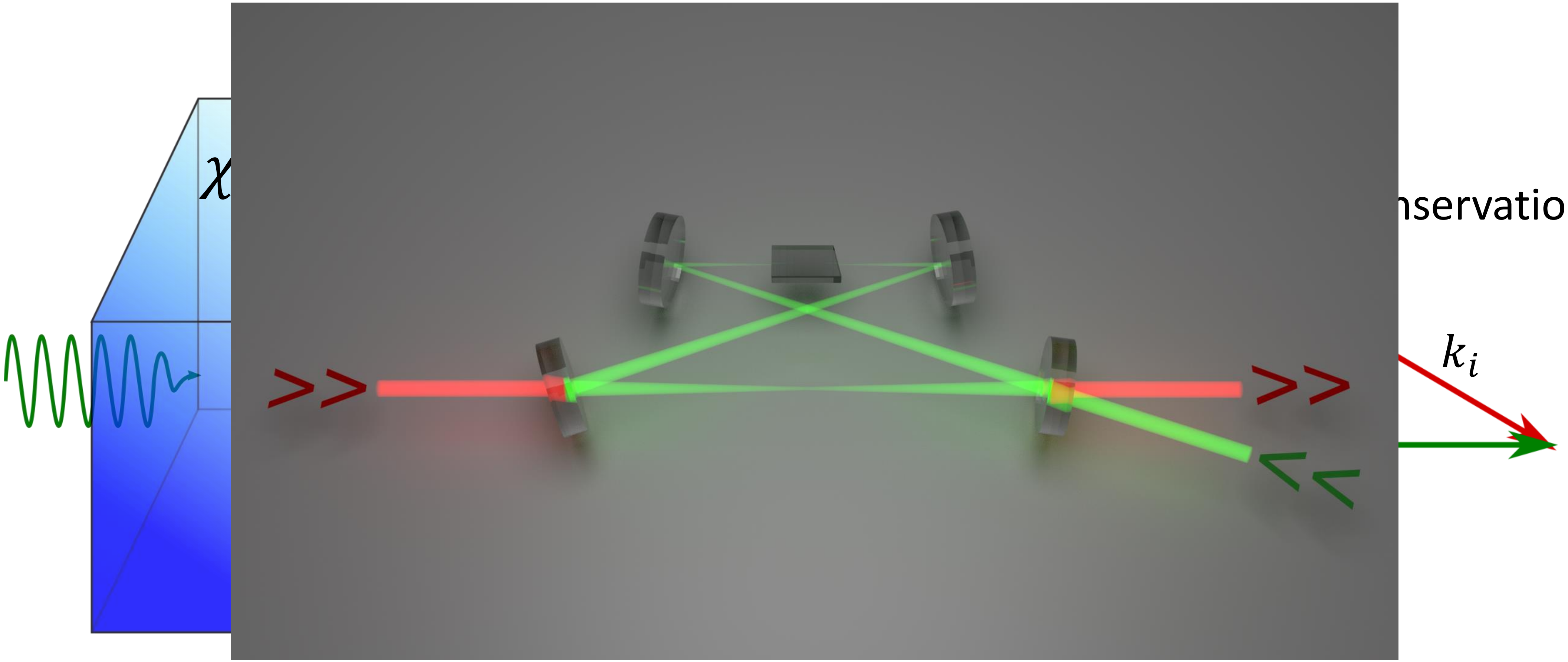


Energy conservation

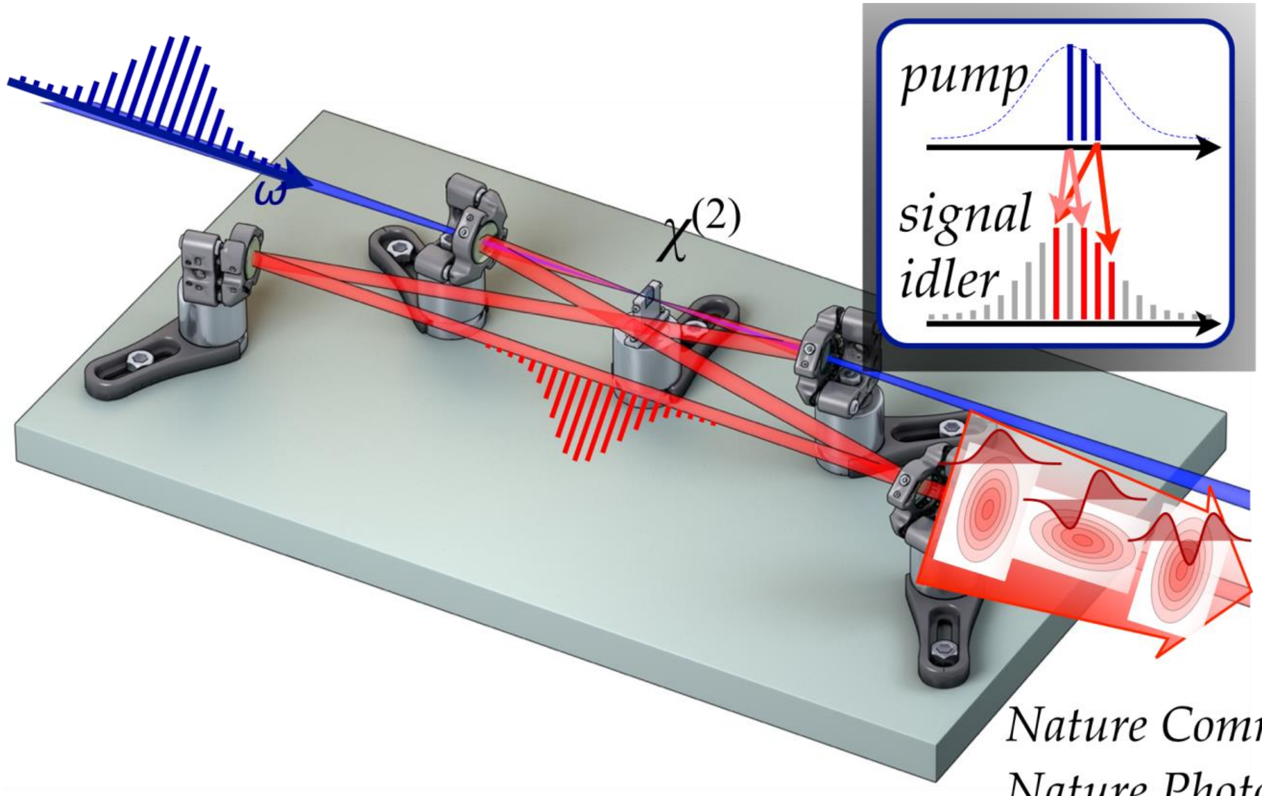


Momentum conservation

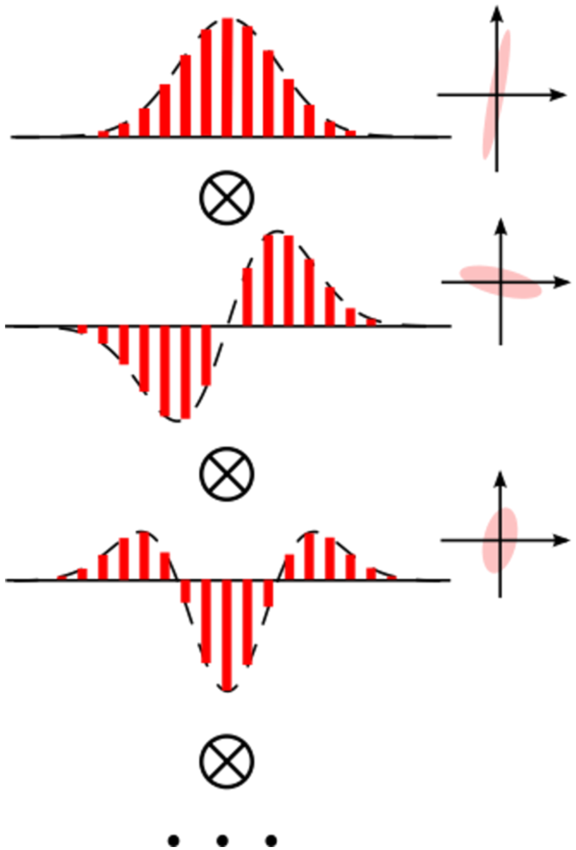



k_i

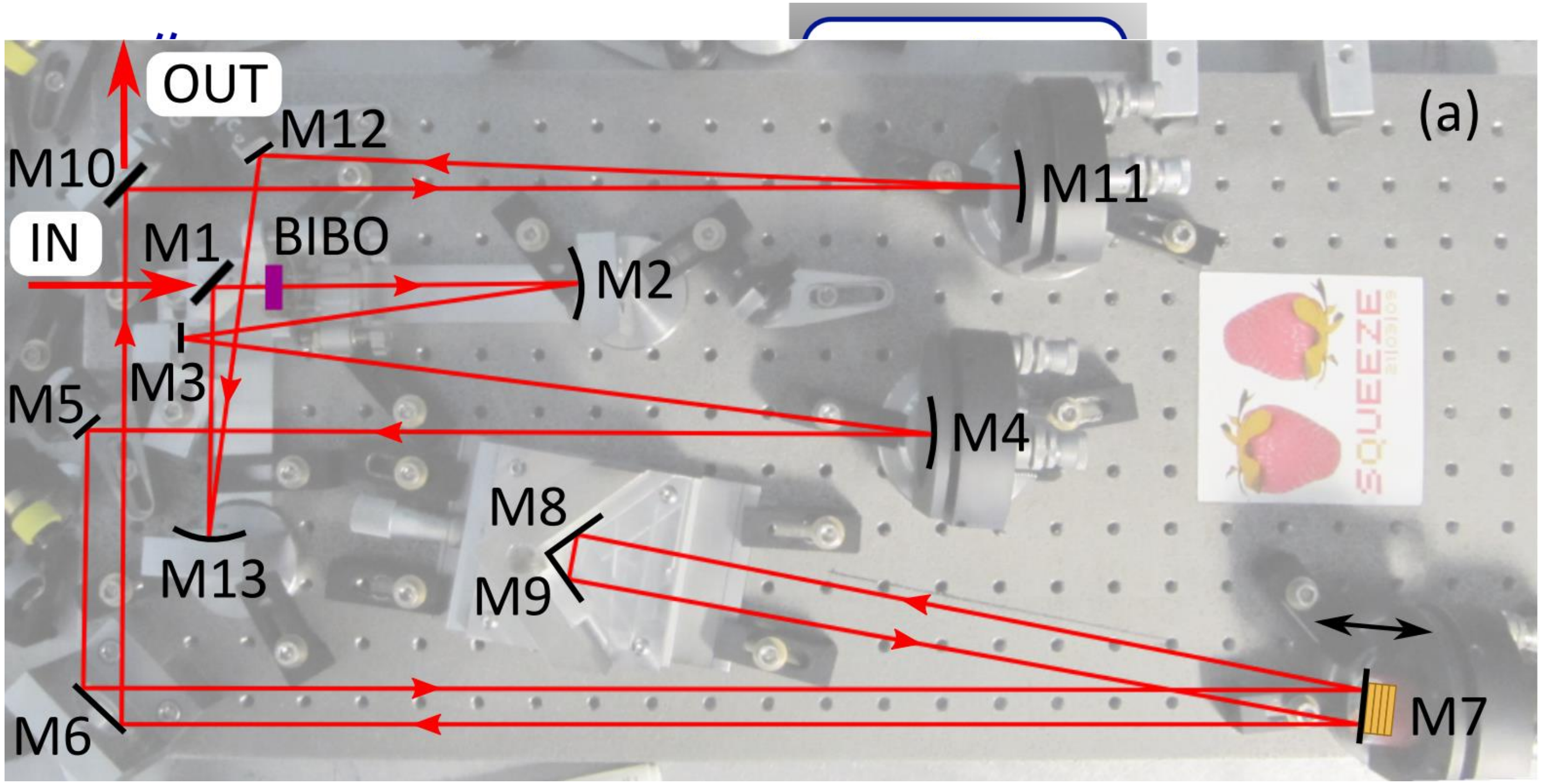
Pump shaping optimization

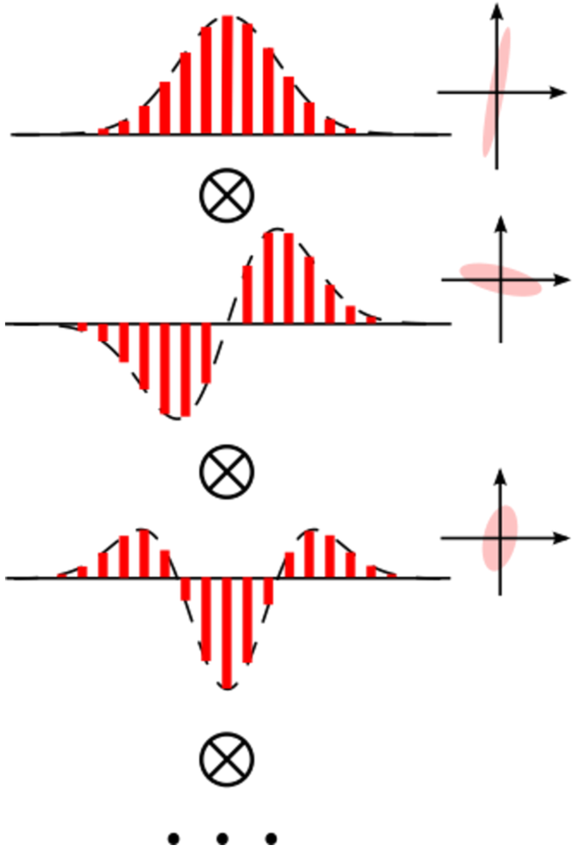


Nature Commun. **9**, 15645 (2017)
Nature Photon. **8**, 109 (2014)



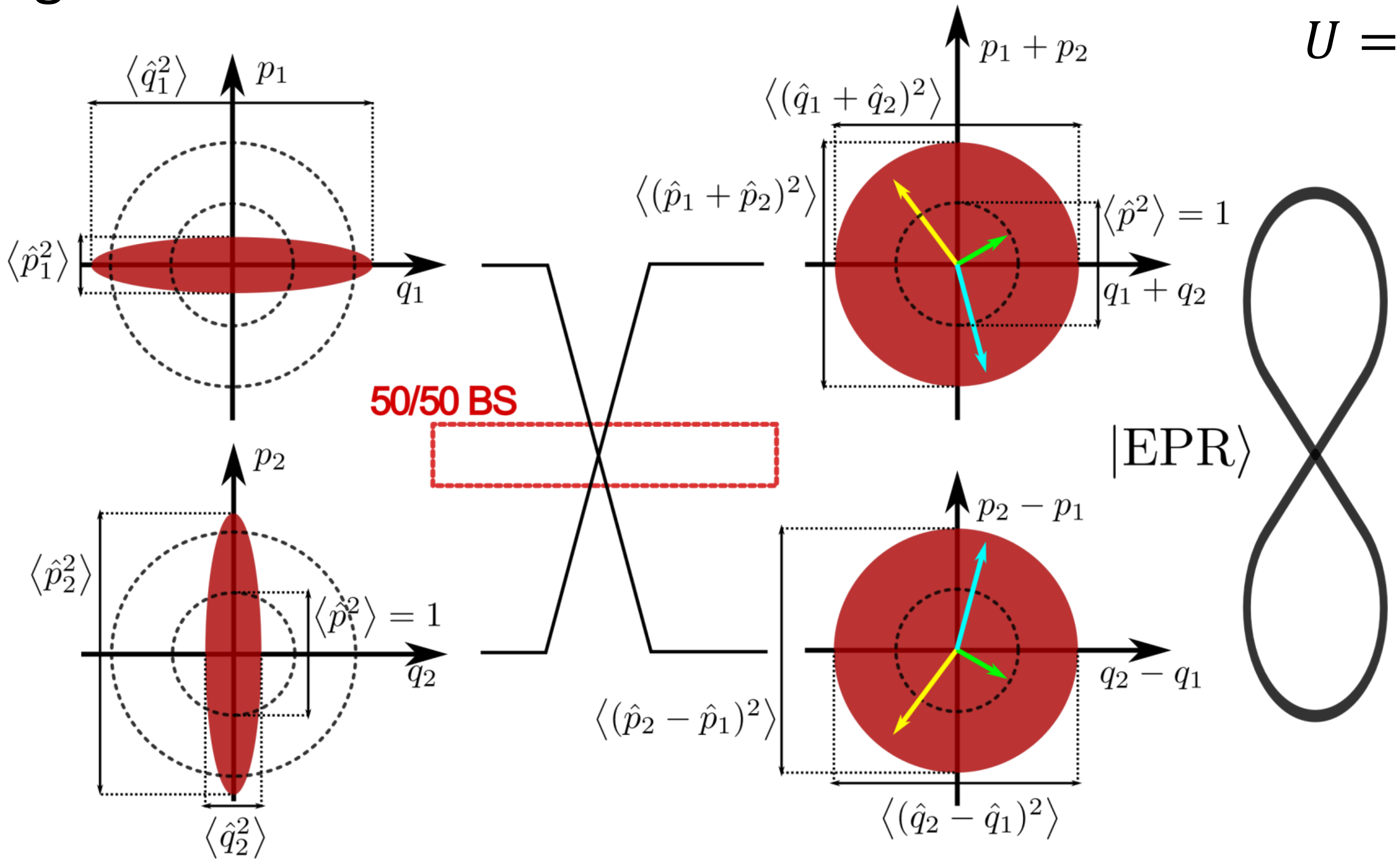
Pump shaping optimization





- CV Entanglement

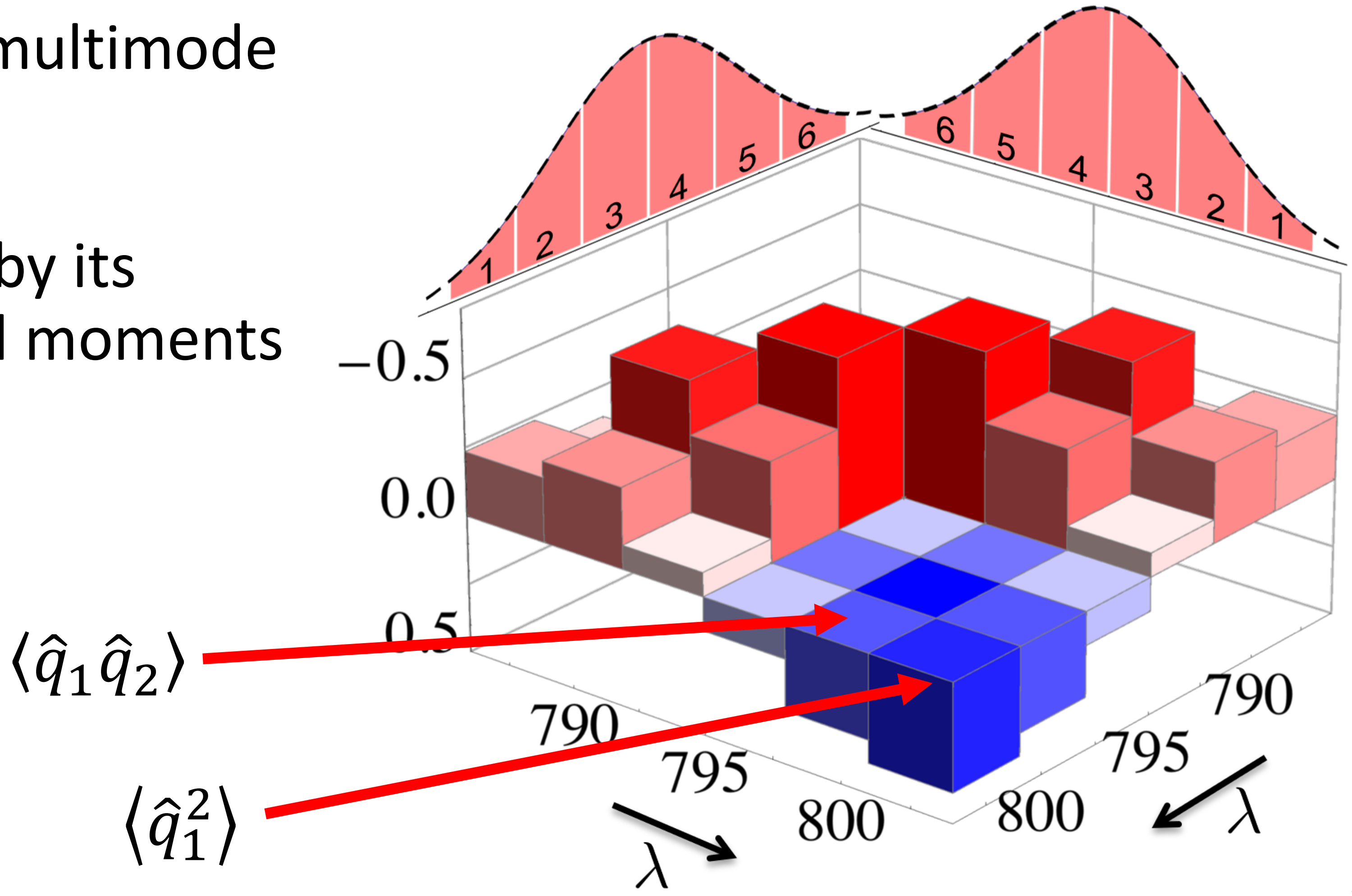
$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



Pump shaping optimization

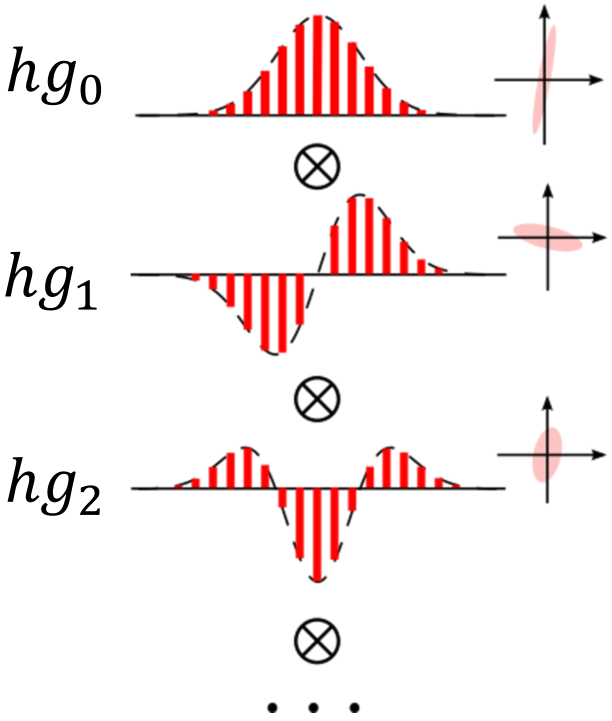
- SPOPO output is a multimode Gaussian state

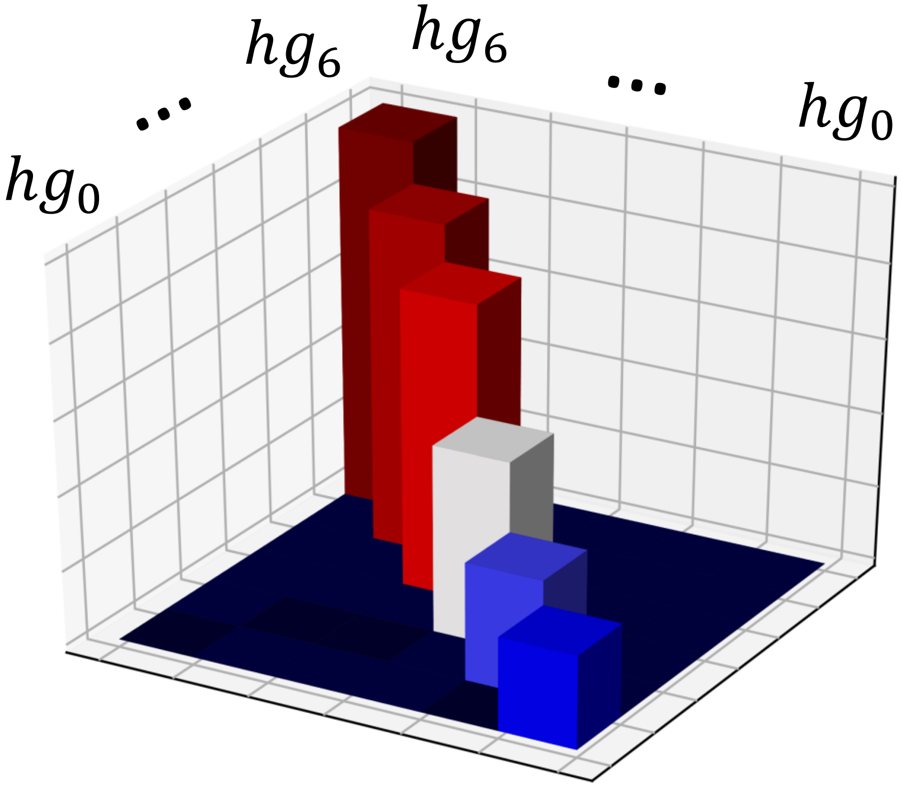
  Fully characterized by its quadratures second moments (covariance matrix)



- SPOPO output is a multimode Gaussian state

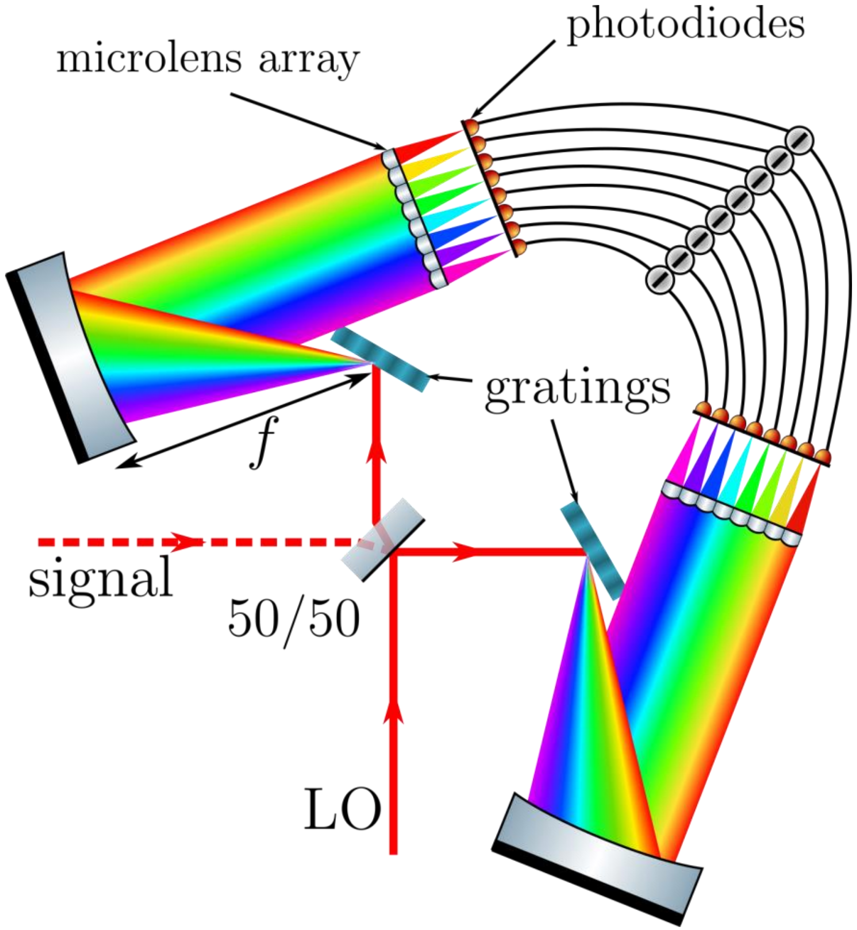
  Fully characterized by it's quadratures second moments (covariance matrix)
  -> In any basis

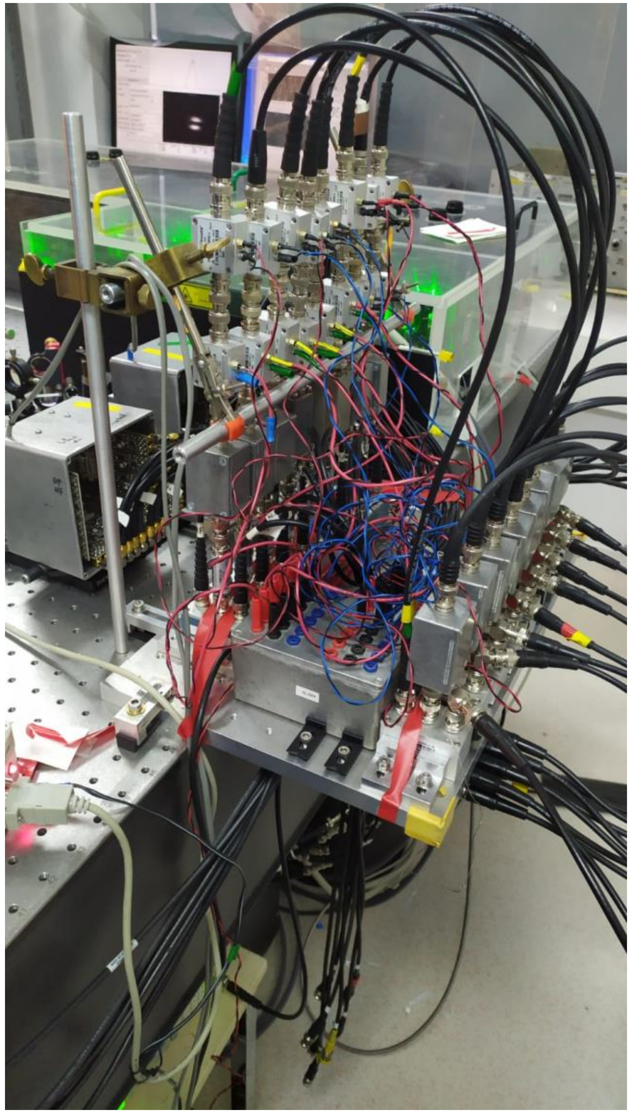




SPOPO

- Measuring the covariance matrix:
  Multipixel Homodyne detection



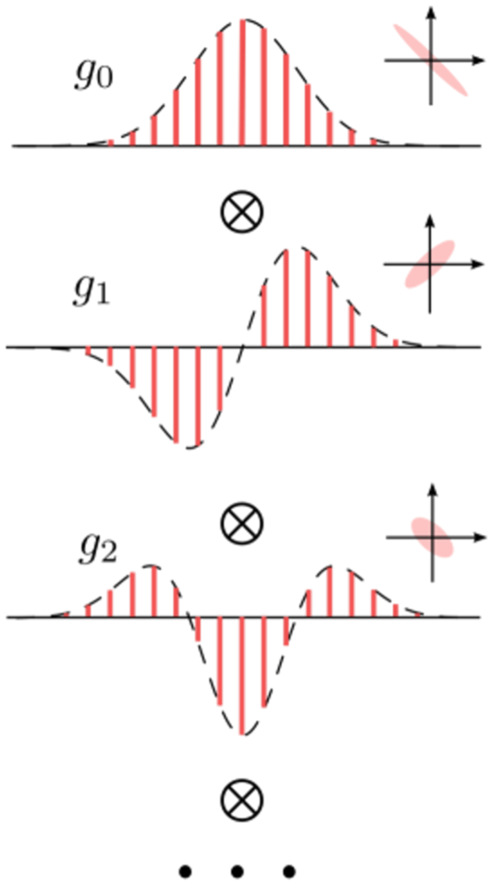


- What to do with this multimode quantum resource

    1) Metrology
        - Better with **all squeezing in one mode** [1]
        - Need to tune the squeezed mode shape

    2) Measurement based quantum computing [2]
        - Better with **identical squeezing amongst all modes**
        - Good to tune mode shape (to match measurement)

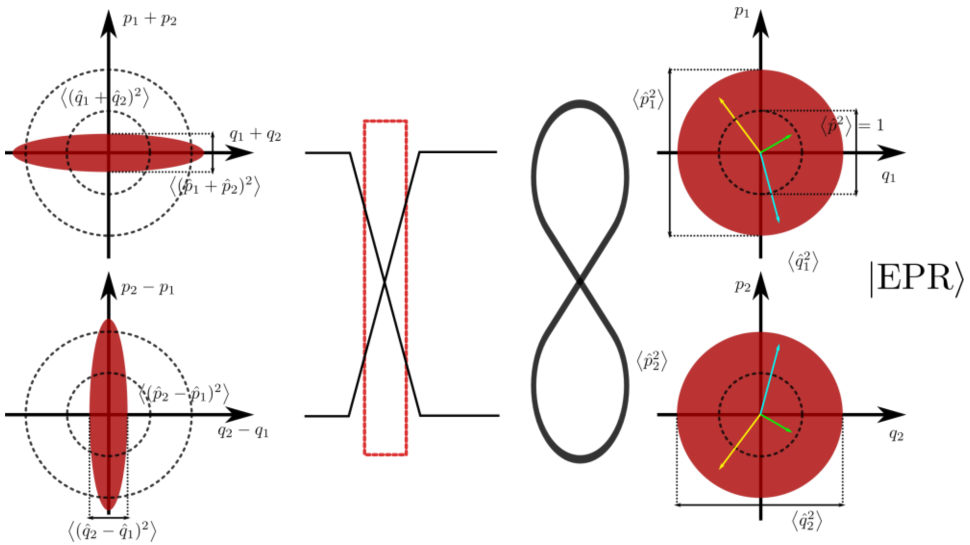
- Both require tunability

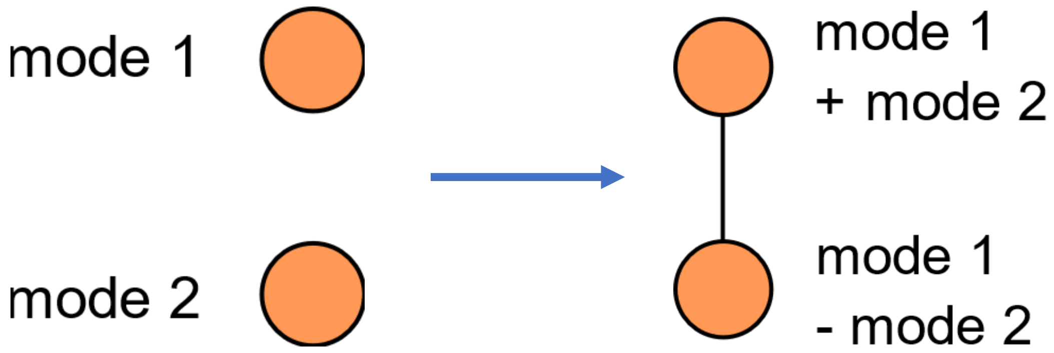


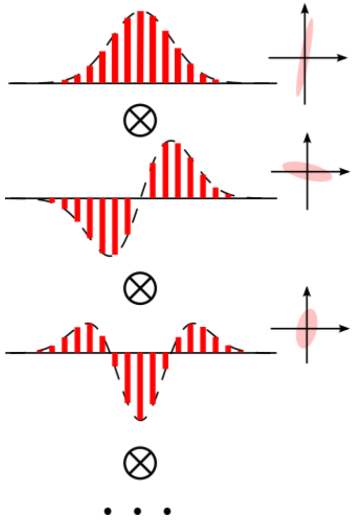
[1] O. Pinel et. al. Ultimate sensitivity of precision measurements with intense Gaussian quantum light: A multimodal approach. Phys. Rev. A 85, 010101(R)

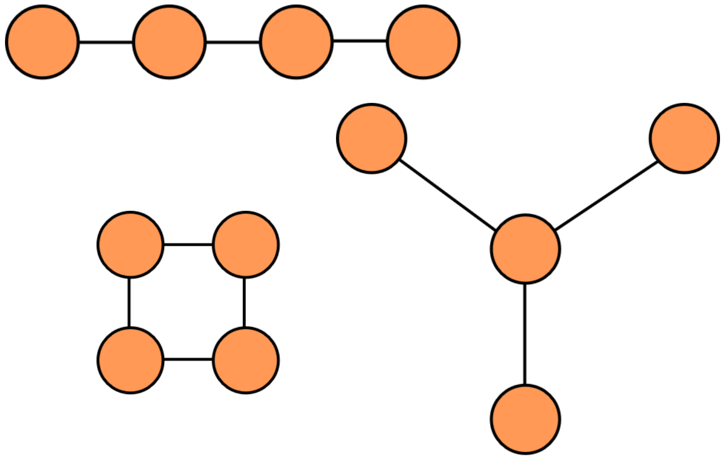
[2] N.C Menicucci et. al. Universal Quantum Computation with Continuous-Variable Cluster States. Phys. Rev. Lett. 97, 110501

Pump shaping optimization





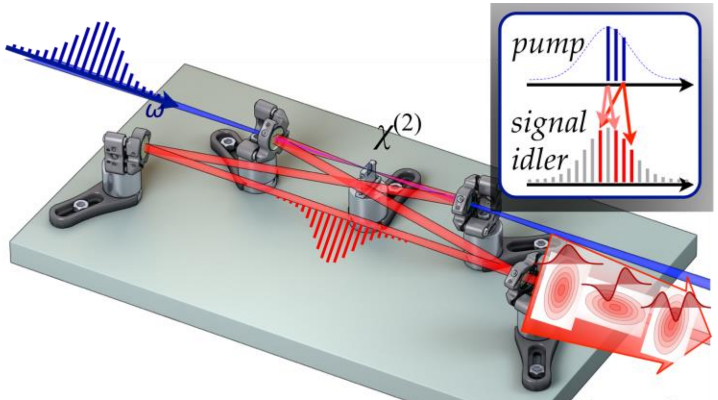




Cluster states

Pump shaping optimization

- How do we tune the output state?



Nature Commun. **9**, 15645 (2017)
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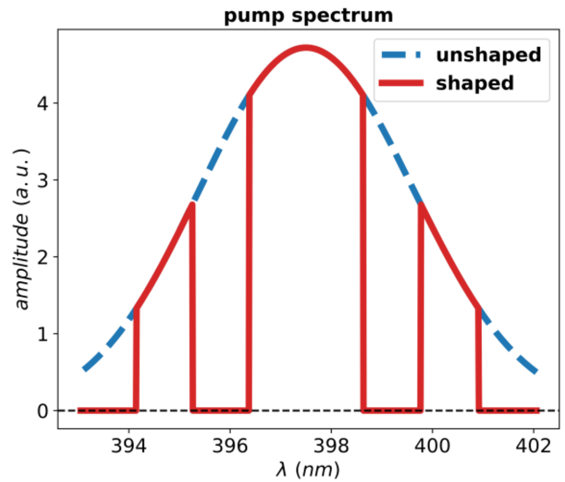
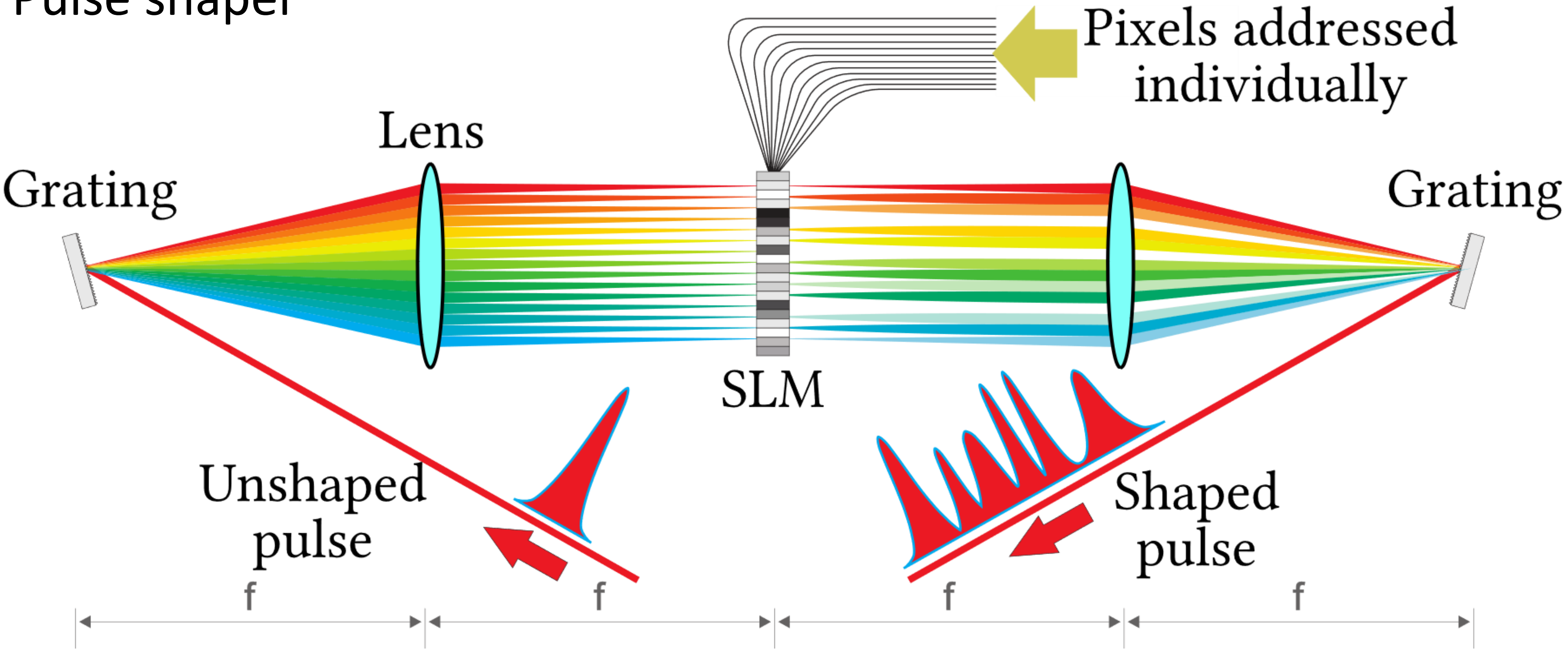
-> Down conversion characterized by $L_{k,l}$:

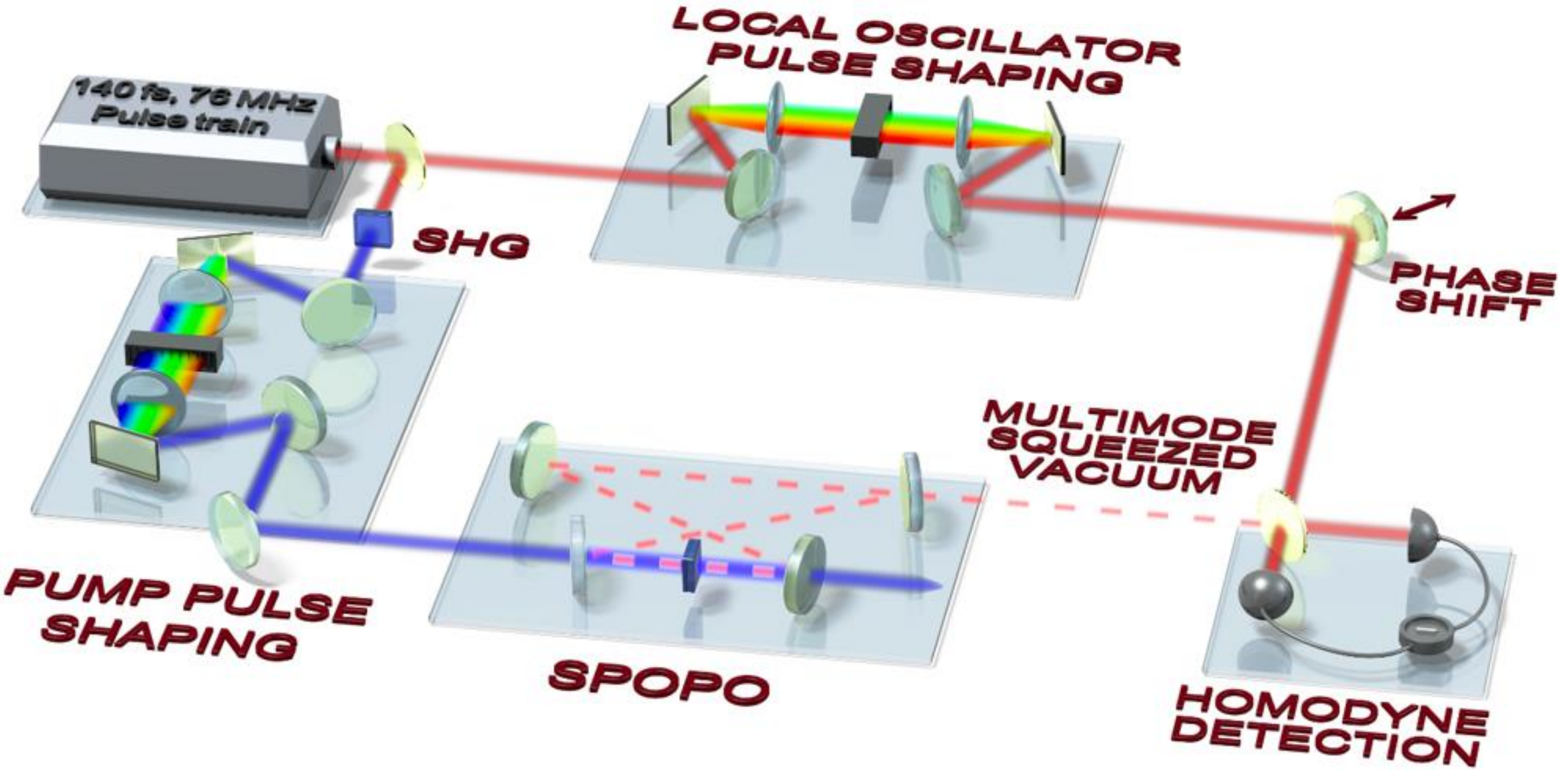
$$L_{k,l} = \mathrm{sinc}\left(\frac{\Delta k(\omega_k,\omega_l) l_c}{2}\right) \alpha_p(\omega_k + \omega_l)$$

Crystal properties **Pump spectrum**



- Pulse shaper

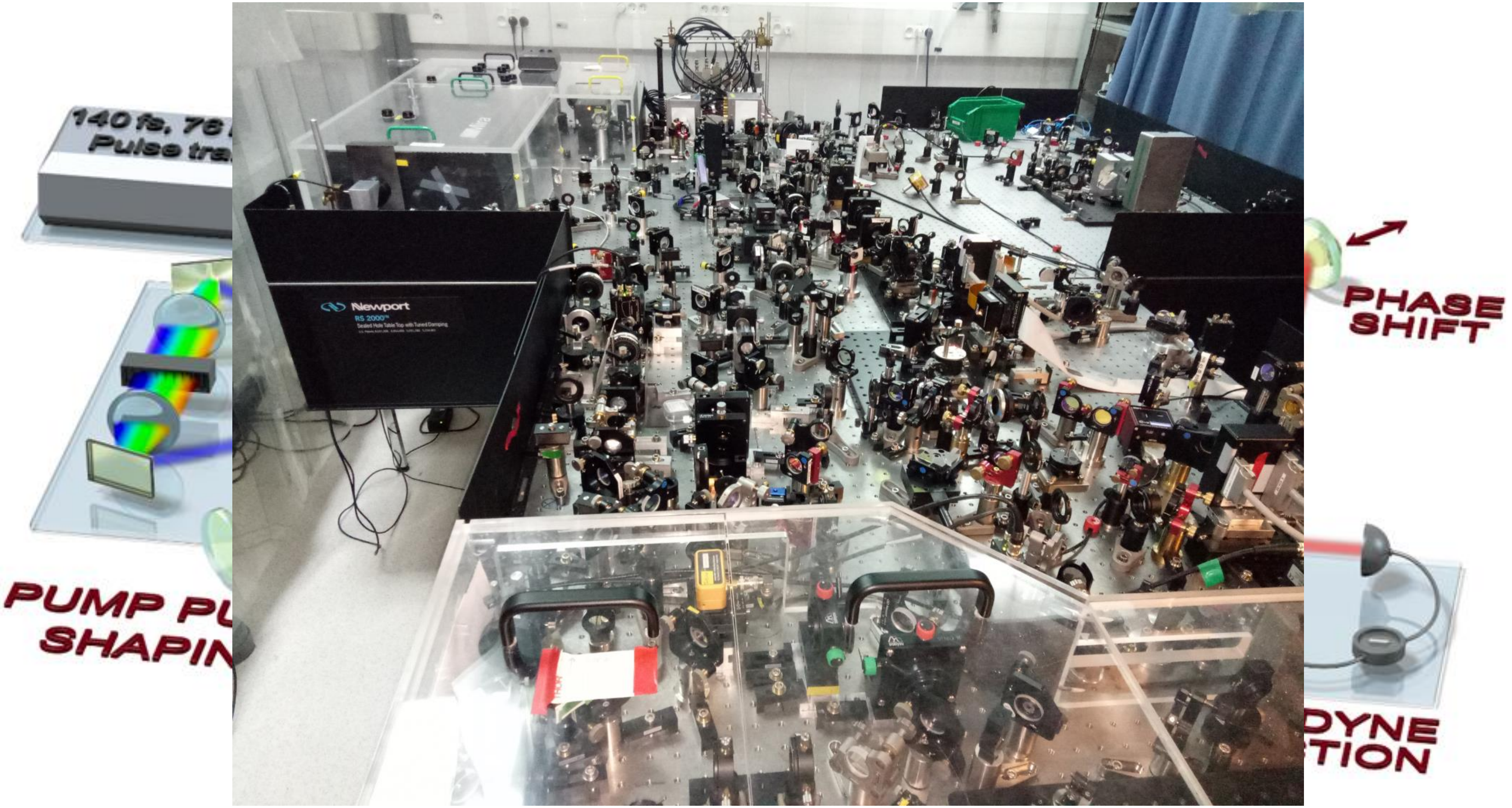



140 fs, 76 MHz
Pulse train
LOCAL OSCILLATOR
PULSE SHAPING
SHG
PHASE
SHIFT
MULTIMODE
SQUEEZED
VACUUM
PUMP PULSE
SHAPING
SPOPO
HOMODYNE
DETECTION

LKB

Pump shaping optimization

Australian National University



- Optimization with simulation
  - To maximize **squeezing in a single mode** -> **metrology** 1
  - To get **equal squeezing** in as many mode as possible -> **measurement based quantum computing** 2
  - Pump divided in **8 frequency pixels**, tunable in **phase** and **amplitude**
  - Two distinct optimization algorithm -> same results

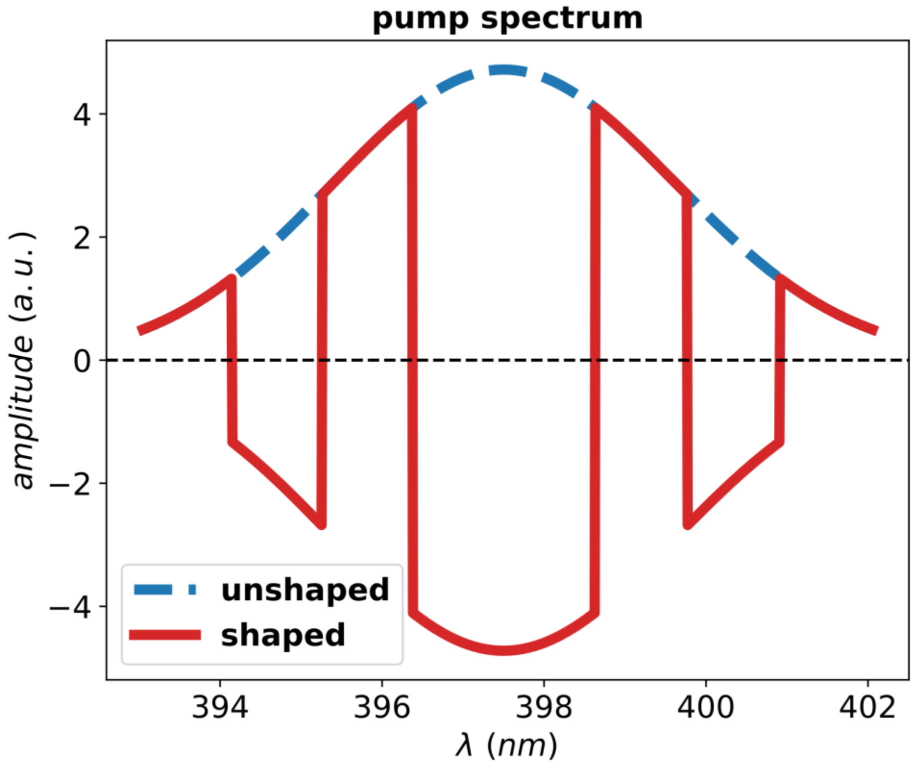
Pump shaping optimization

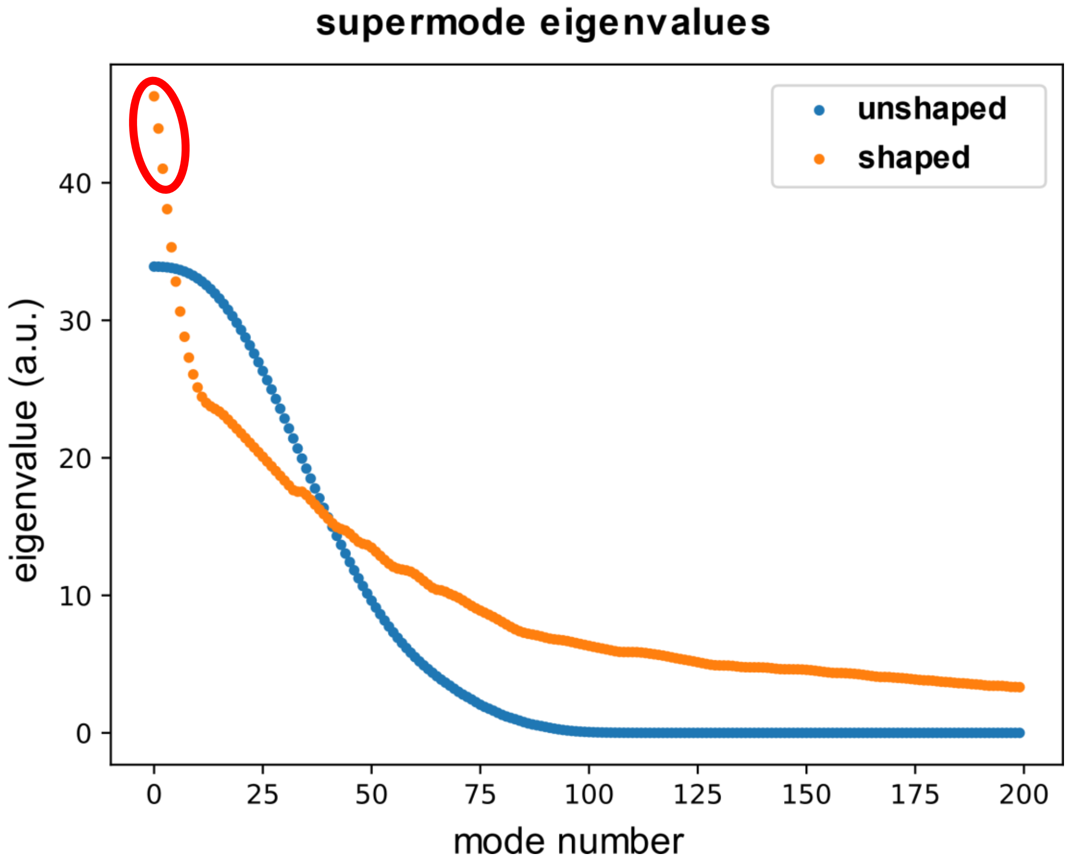
LKB

Australian National University

1

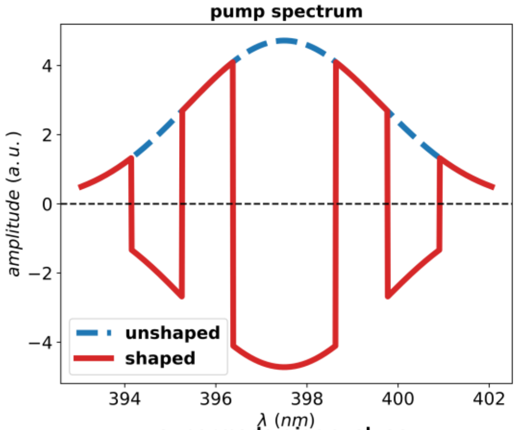
- Results: maximum squeezing

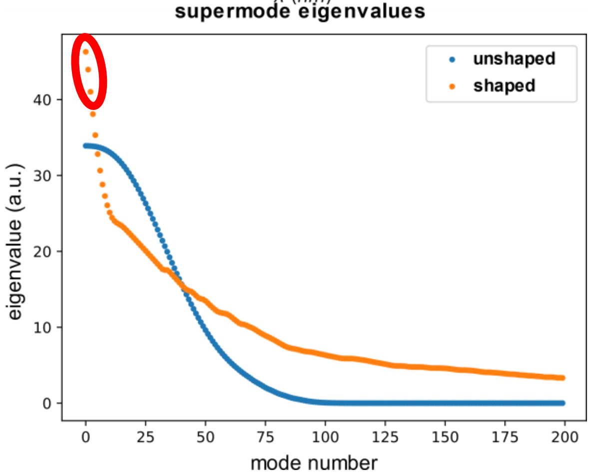


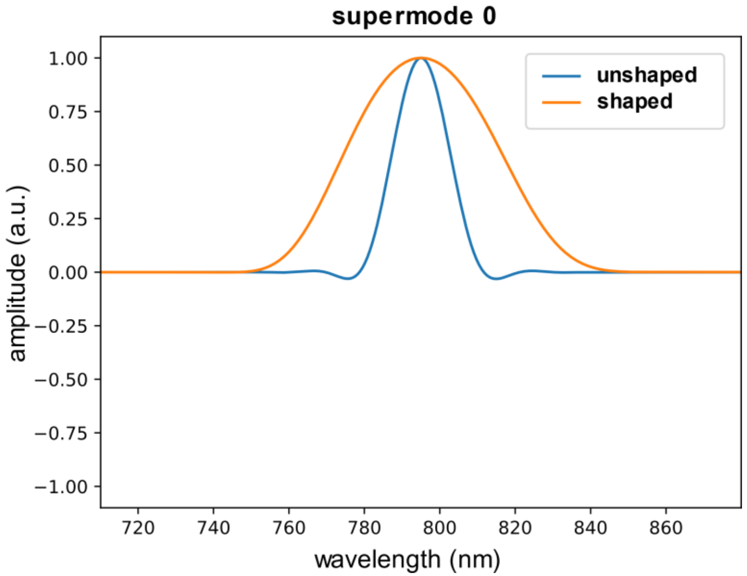


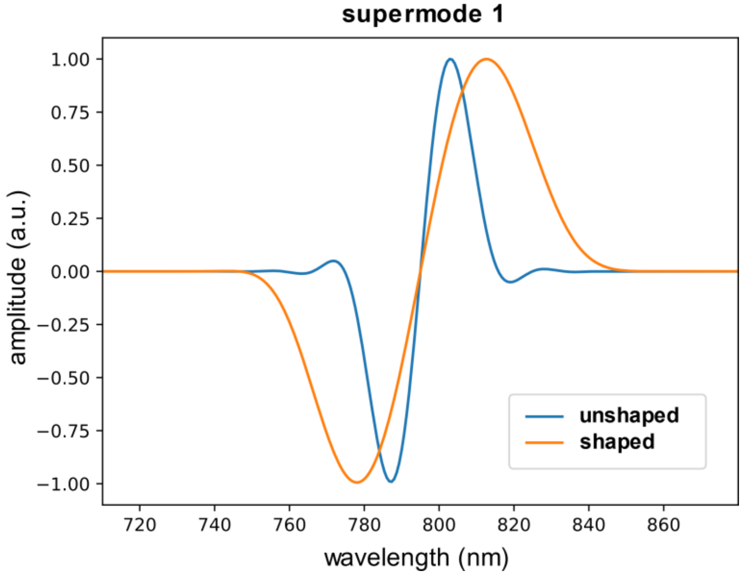
Pump shaping optimization

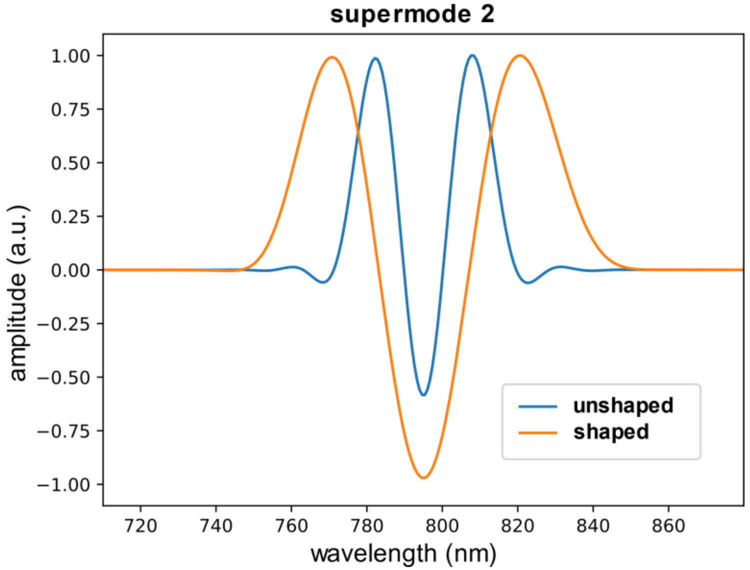
- Results:
  maximum squeezing







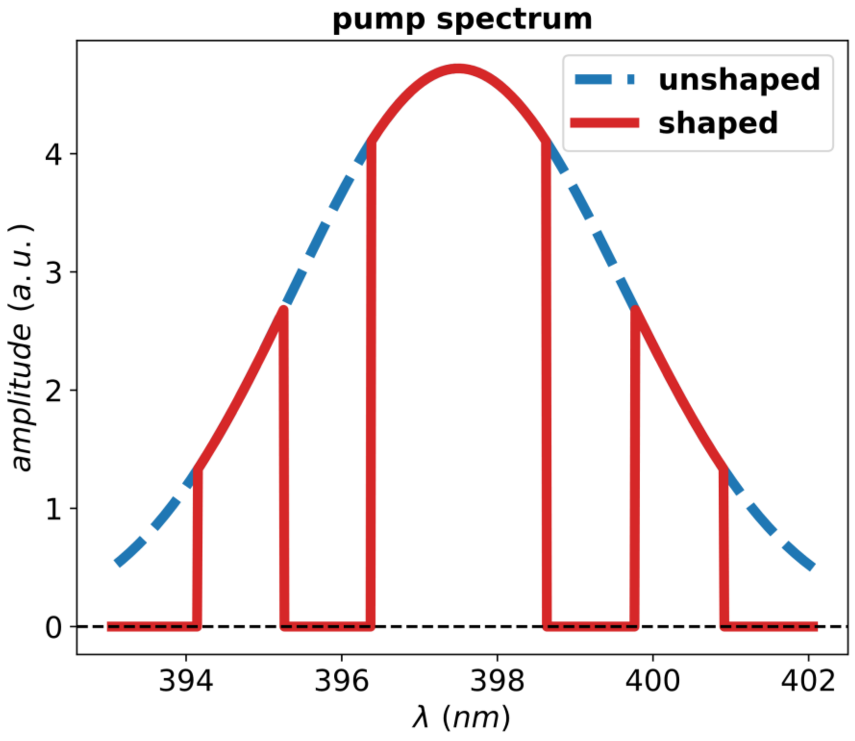


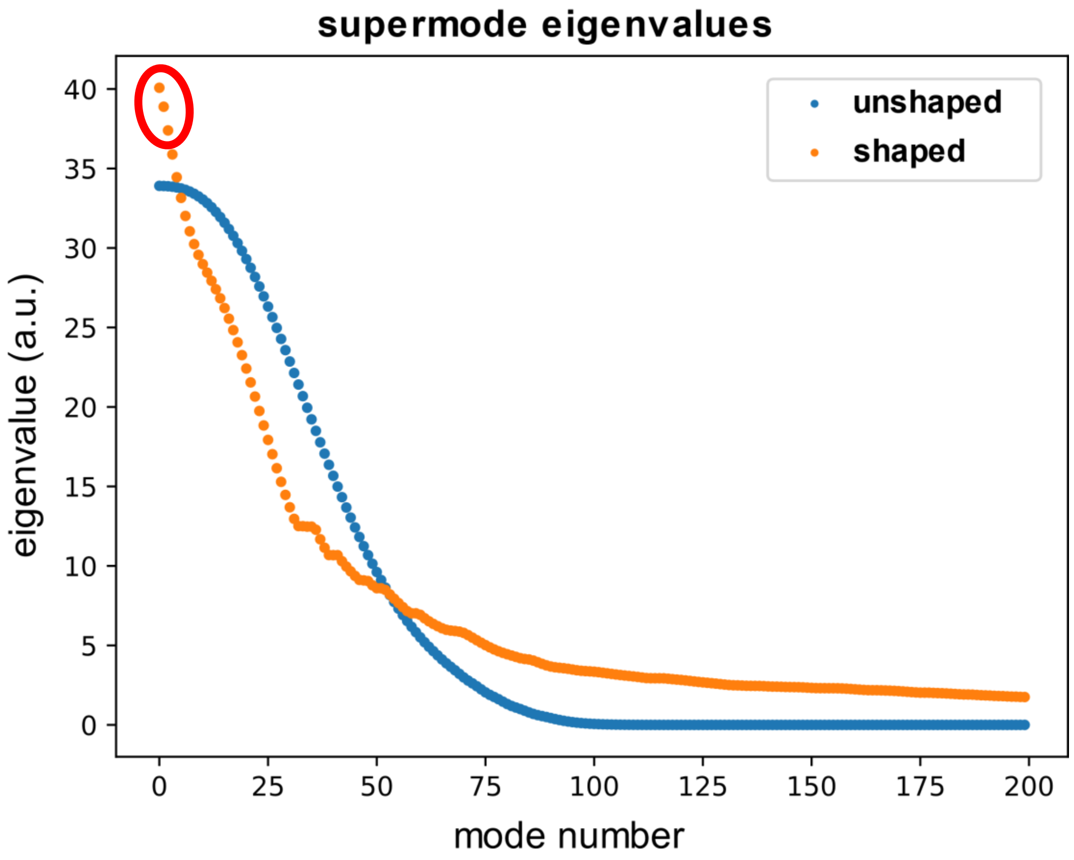


1

1

- Results: maximum squeezing (amplitude shaping)

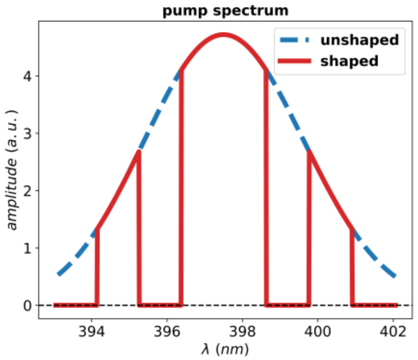


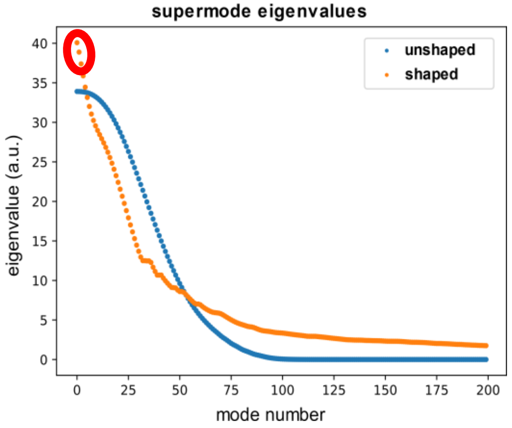


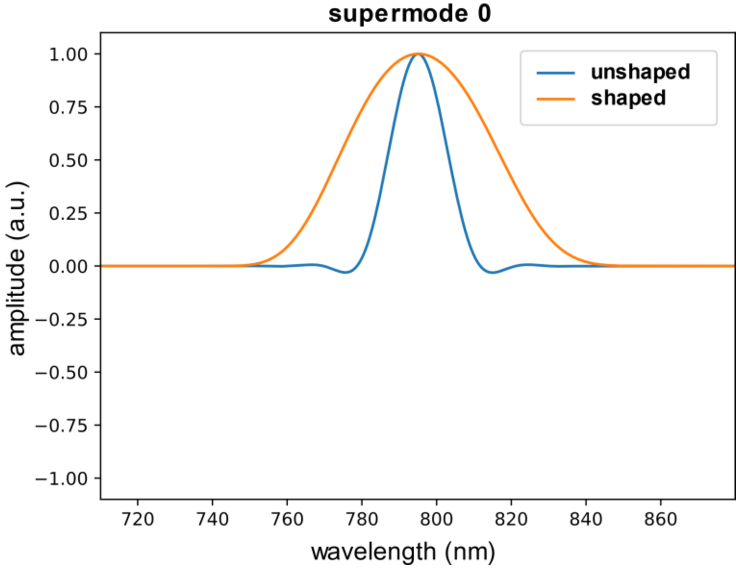
Pump shaping optimization

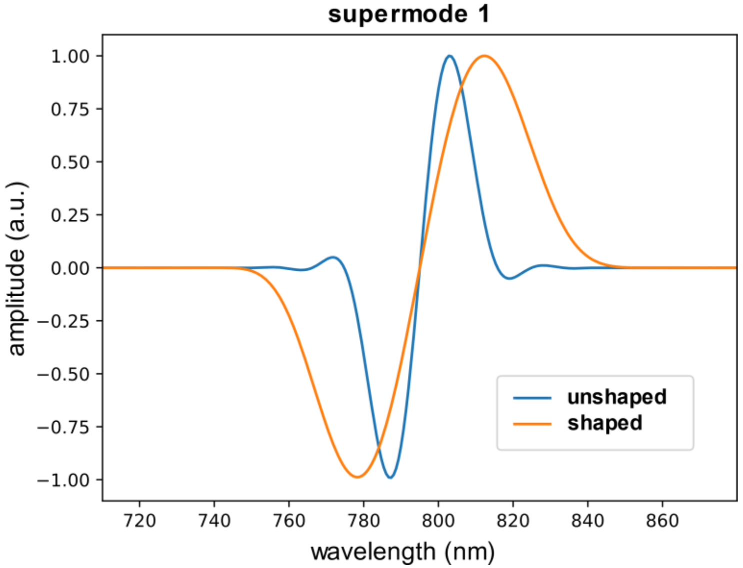
1

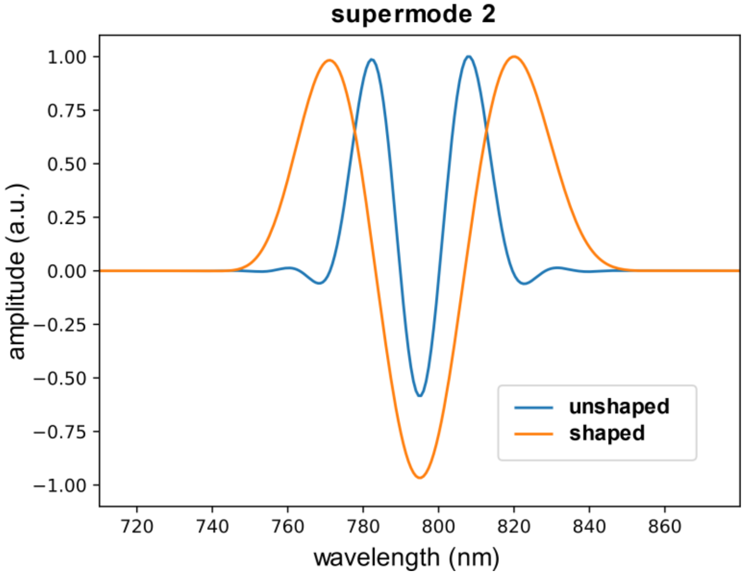
- Results:
  maximum squeezing
  (amplitude shaping)





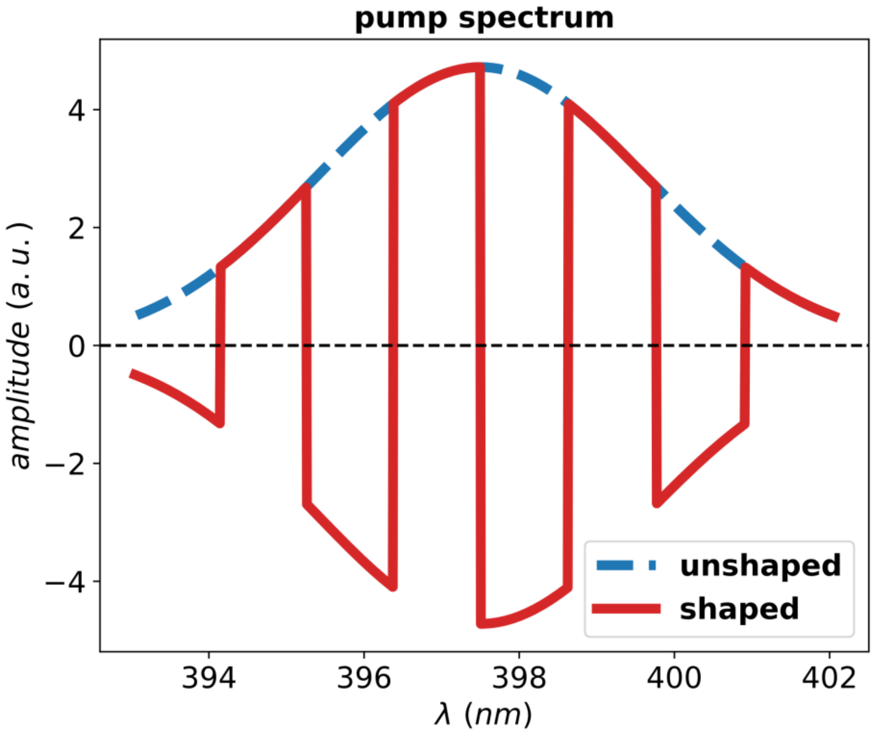


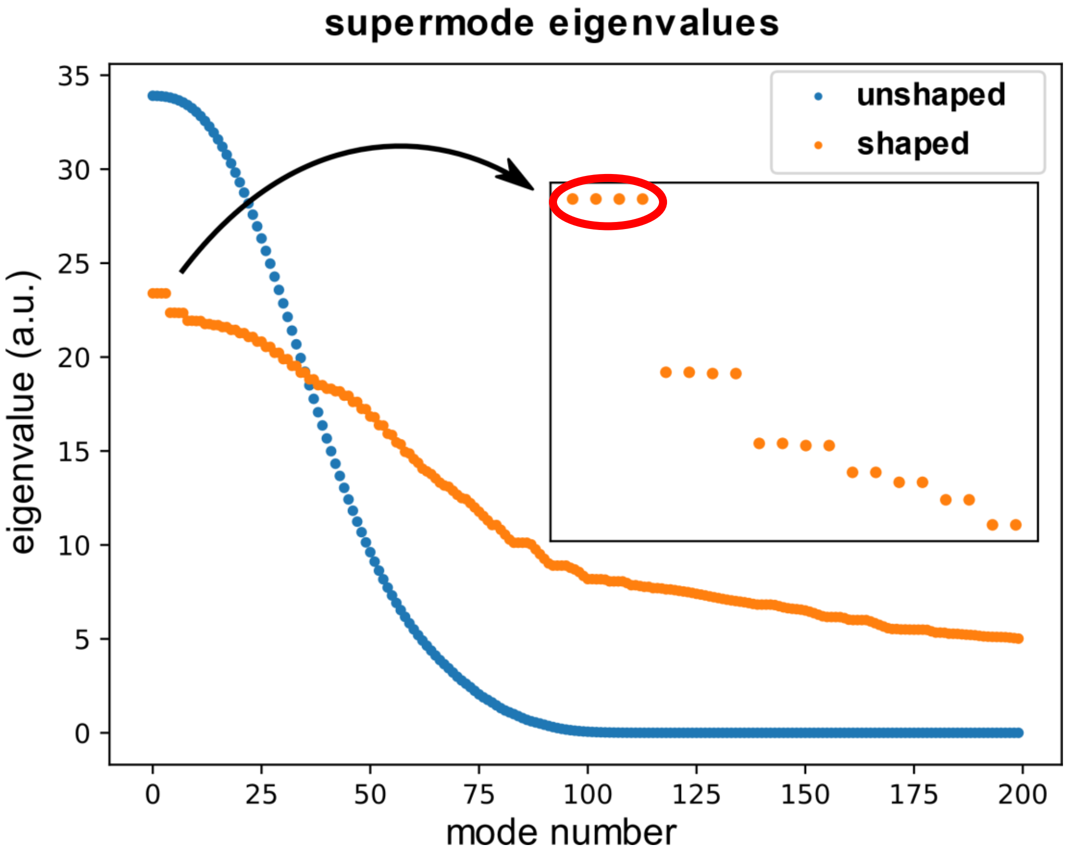




2

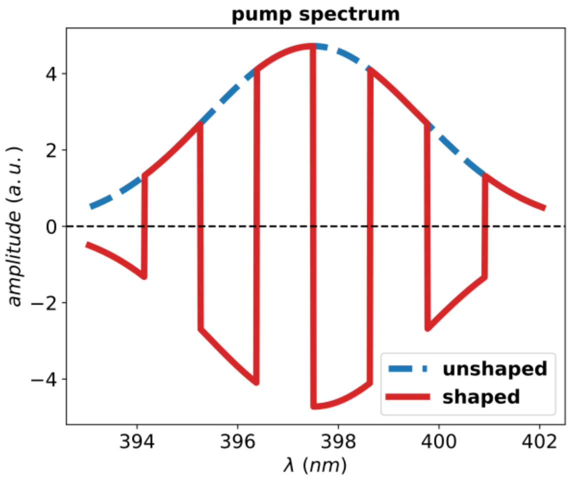
- Results: flat squeezing

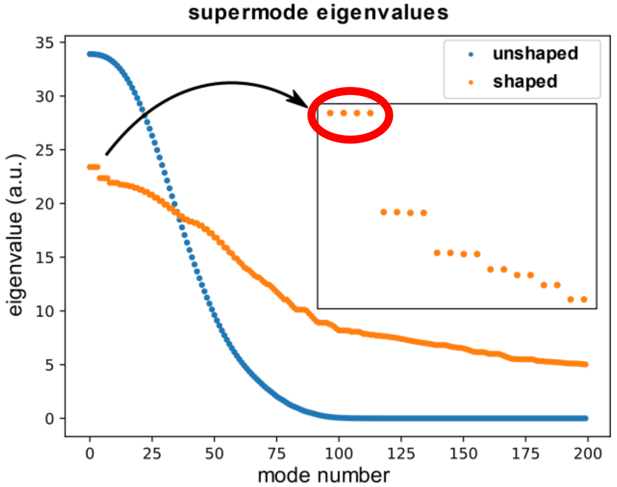


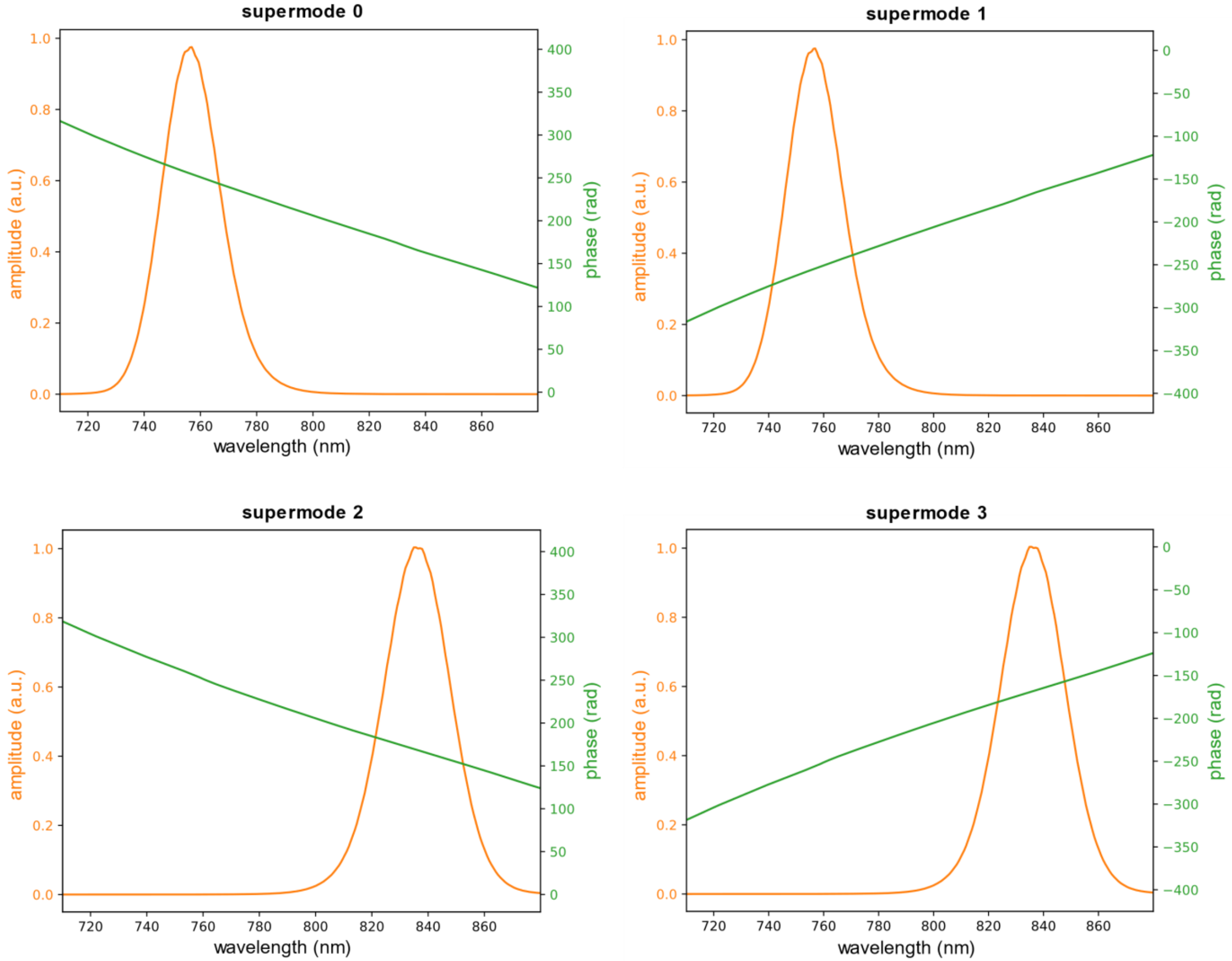


- Results: flat squeezing

2



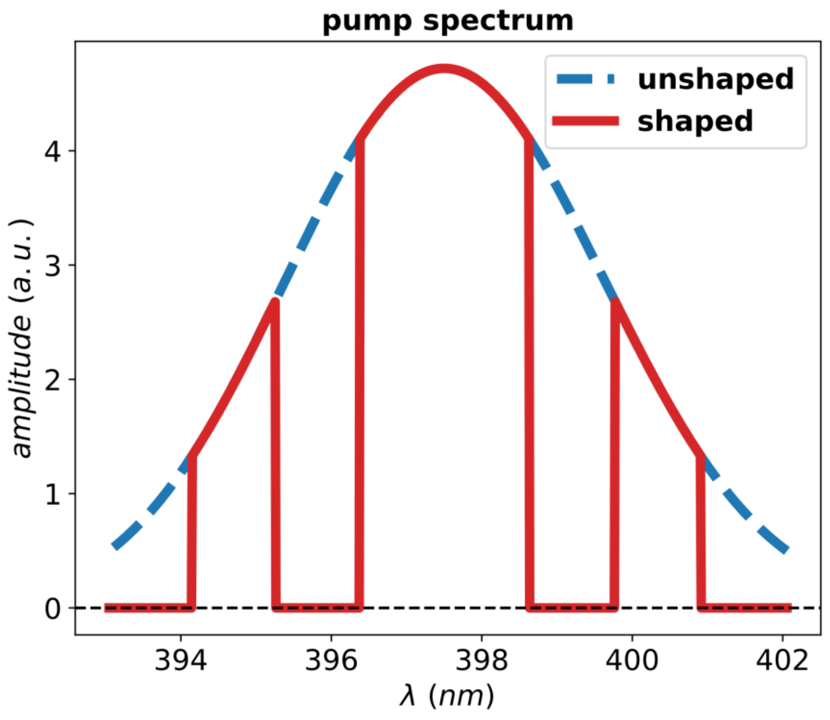


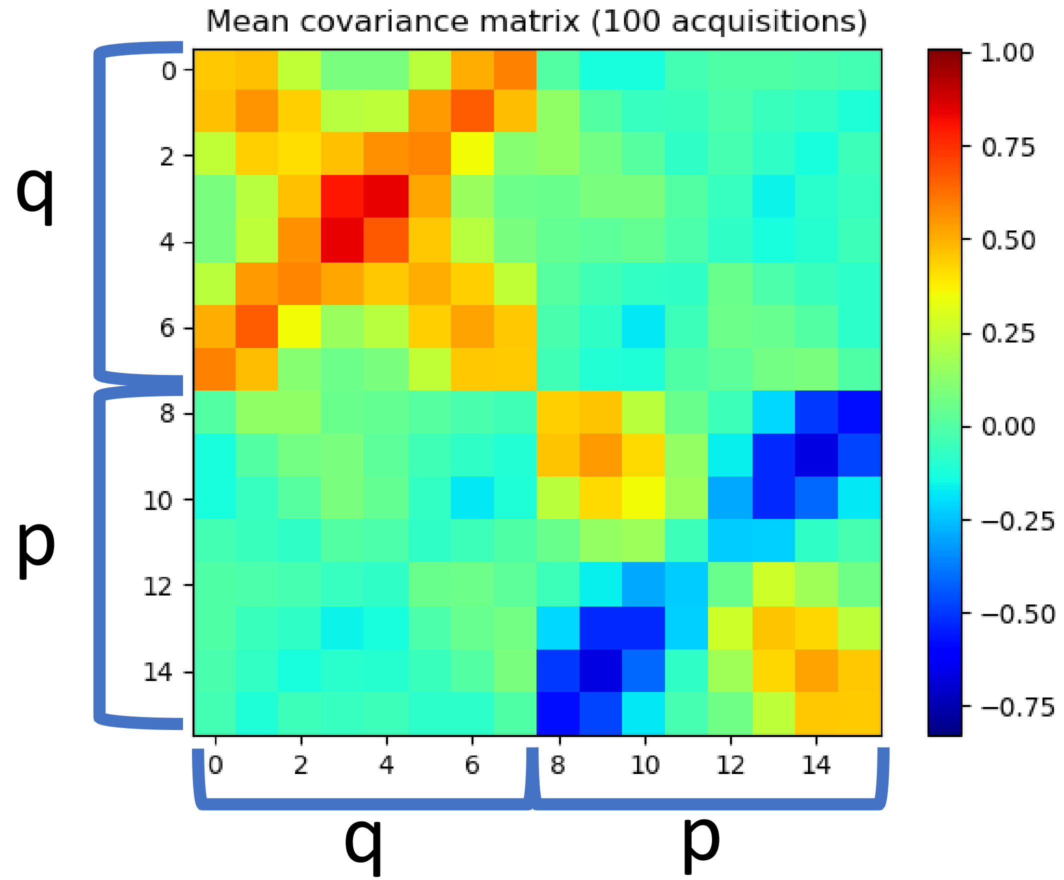


Pump shaping optimization

- Experimental results:
  pump for max squeezing (amplitude shaping)

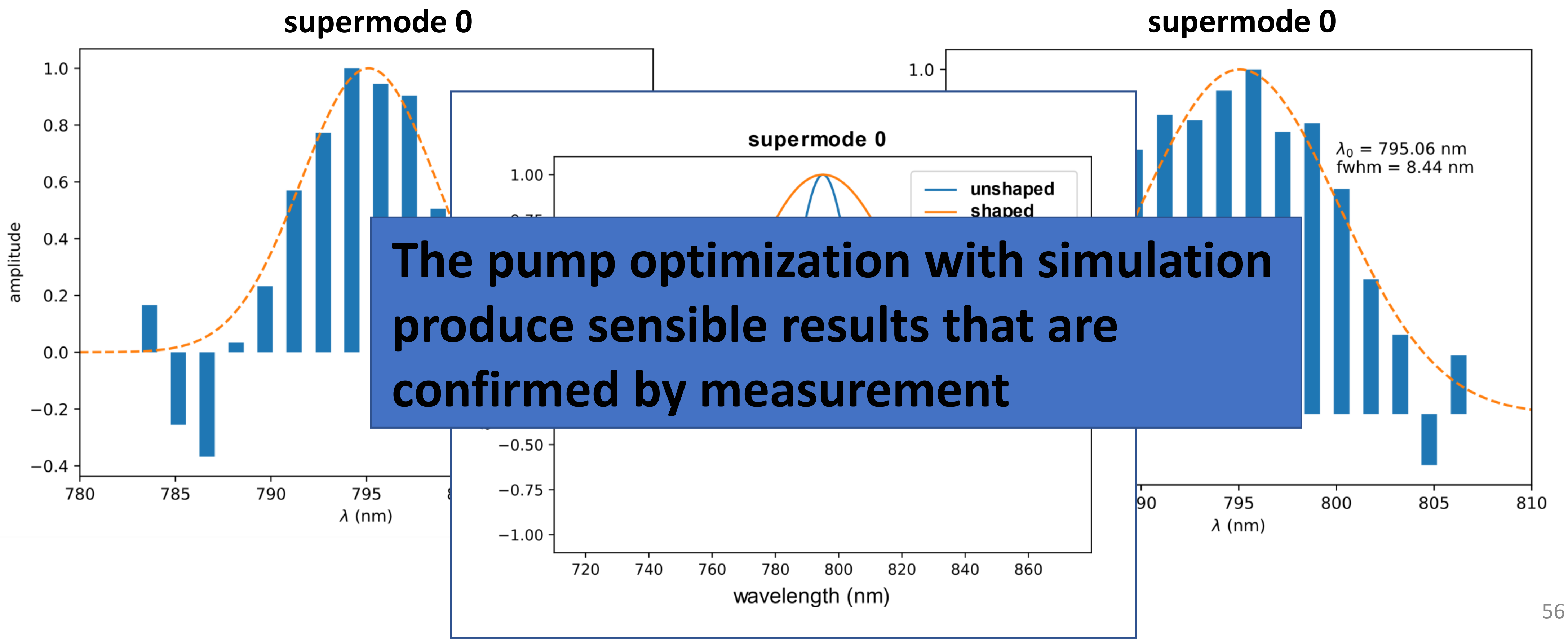
1





- Measure with standard homodyne, 16 frequency bands:
  - We observe the **broadening** of the modes, they keep an HG shape

1

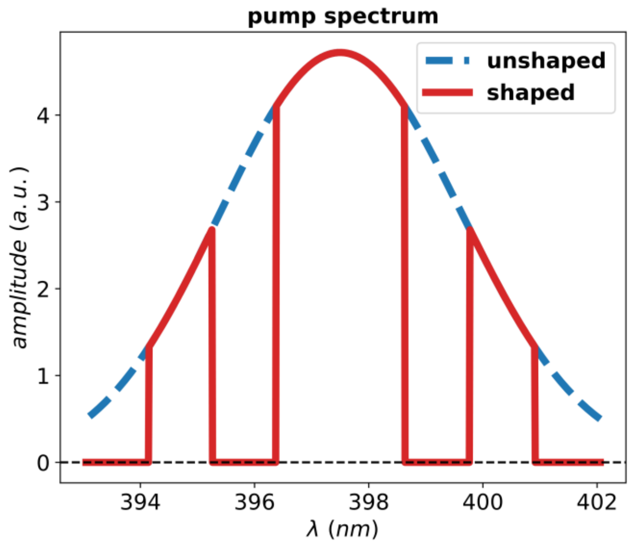


The pump optimization with simulation produce sensible results that are confirmed by measurement

Pump shaping optimization

- Effect of pump shaping on squeezing:
  (pump for max squeezing, amplitude shaping)

1

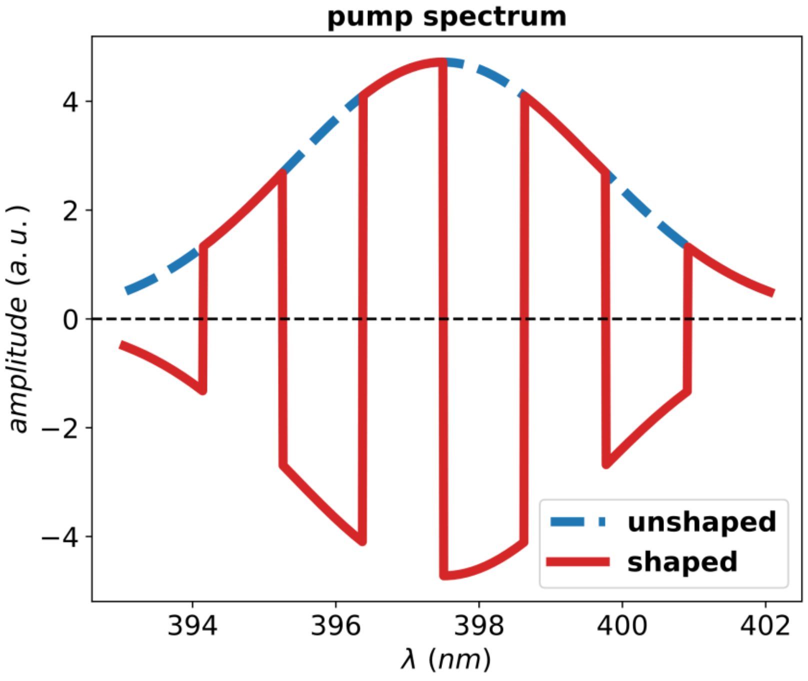


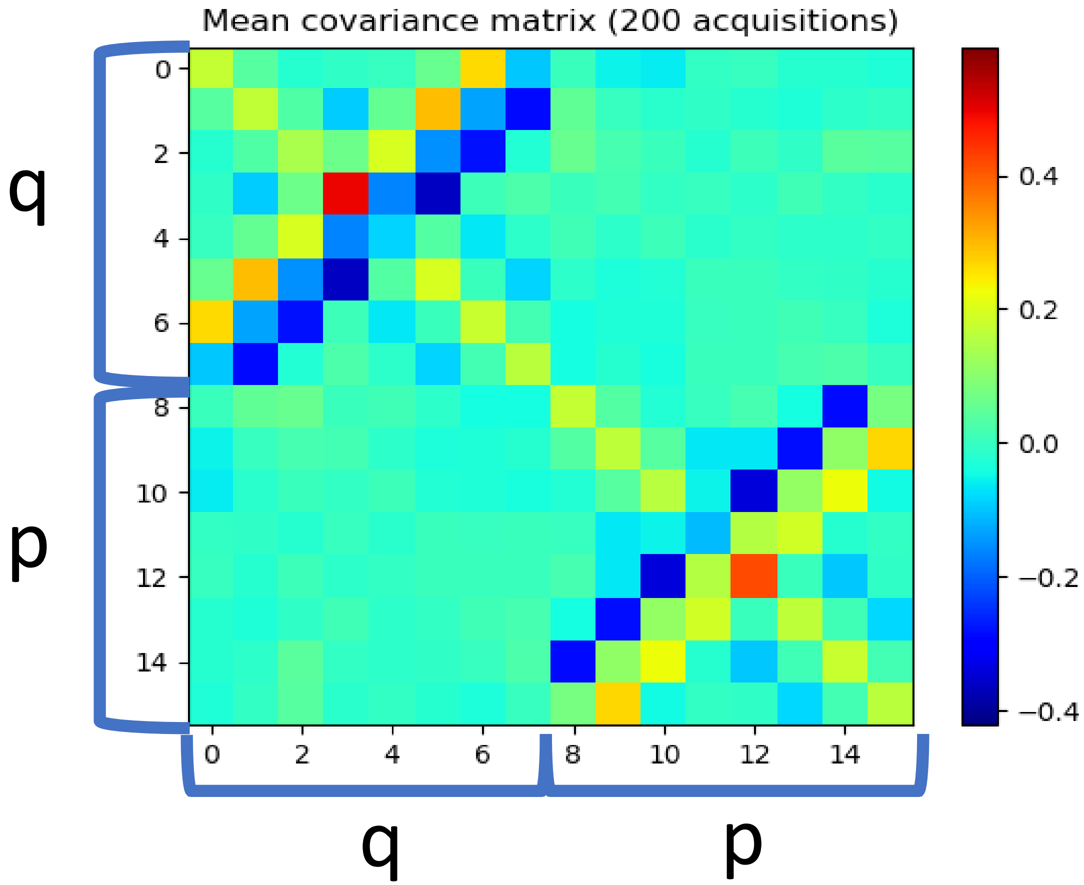
MODE	Squeezing level (dB)		Anti-squeezing level (dB)	
	No pump shaping	Pump shaping	No pump shaping	Pump shaping
HG0	3.79	3.78	4.32	4.55
HG1	1.30	1.81	3.14	4.06
HG2	0.70	0.16	2.81	3.06
HG3	0.63	0.53	1.83	2.72

Pump shaping optimization

- Experimental results:
  pump for flat squeezing, phase shaping

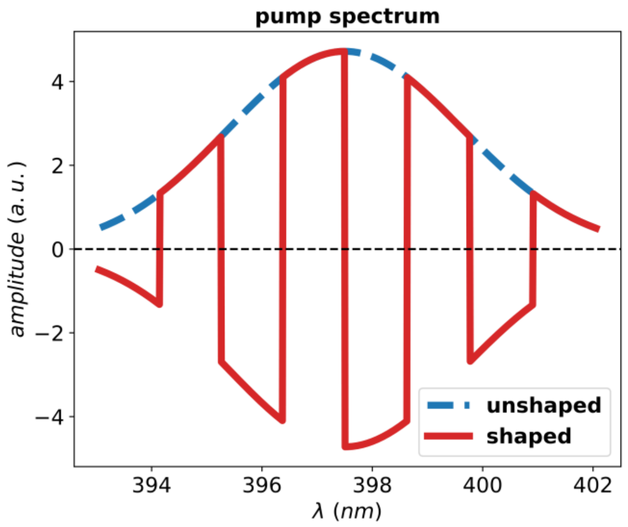
2

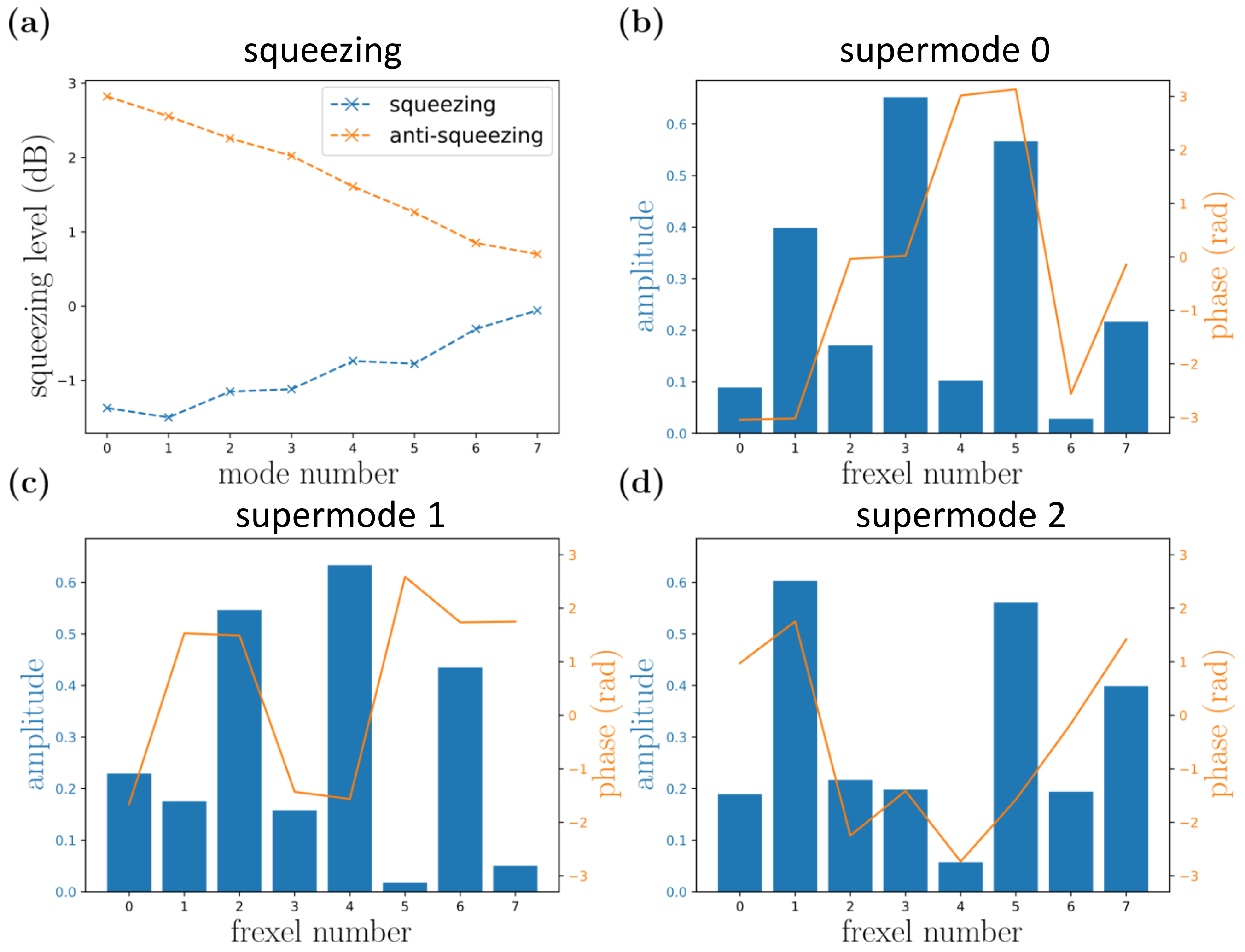




- Experimental results: pump for flat squeezing, phase shaping

2





Conclusions

- **Optimal shapes** of pump (16 dof) for 2 cases where **found by simulation**.
- Partial experimental confirmation
- Results for **case 2** show **intracavity-dispersion** cannot be neglected

Perspective

- More **direct squeezing measurement** to confirm the squeezing improvement (with different LO bandwidth).
- Re-run of the **optimization** with a model that accounts for **intracavity dispersion**.
- Compare with previous pump optimization results using evolutionary algorithm [1]
- **Direct optimization** of experimental set-up

[1] F. Arzani et. al. Phys. Rev. A 97, 033808 (2018)

The End

Thank you !