Coupled nanoresonators

Guilhem Madiot
Institut de Physique de Nice
CNRS, Université Côte d'Azur, Nice, France

guilhem.madiot@univ-cotedazur.fr

Saint-Malo: a giant resonator

Tidal resonance: amplification of tide waves by the local topography

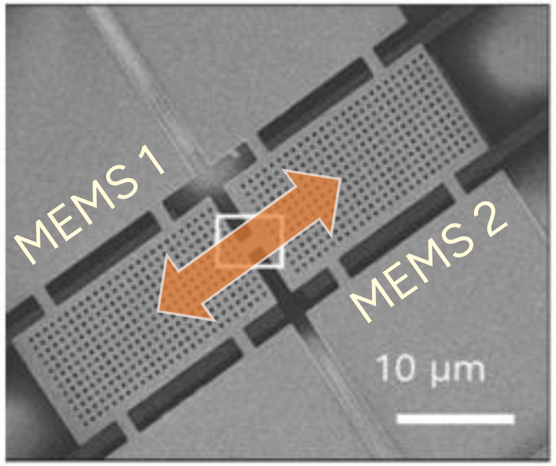
+6h12

Bay of Saint Malo: 10,7 m average differential height of the water level

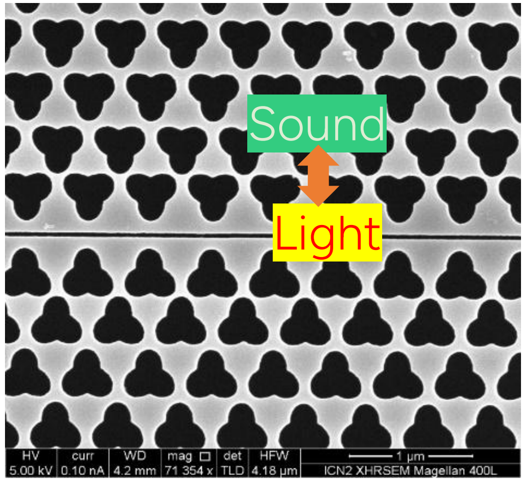
(record of 14,15 m on March 21st, 2015)

Coupling resonators at the nanoscale

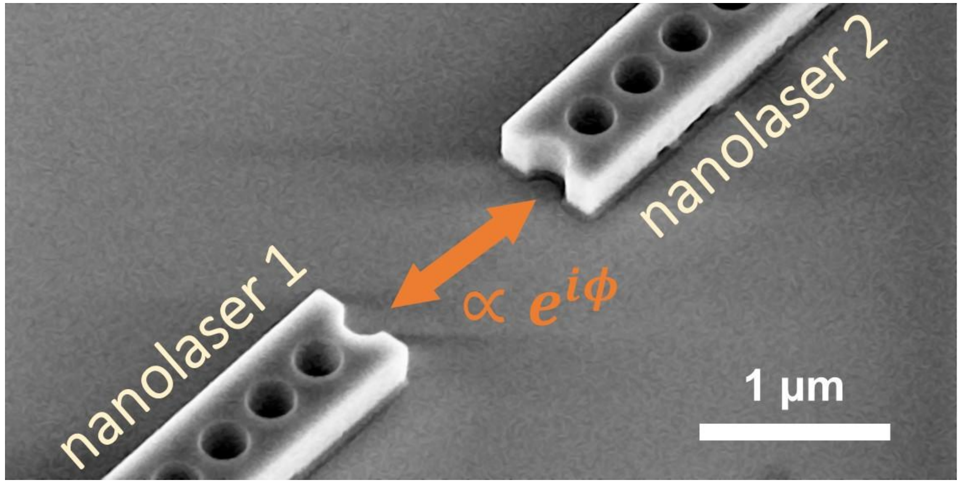
mechanical arrays



Cavity optomechanics



nanolaser arrays



Nonlinear dynamics

Non-Hermitian physics

Motivation : Fermi-Pasta-Ulam-Tsingou « experiment »

Who?

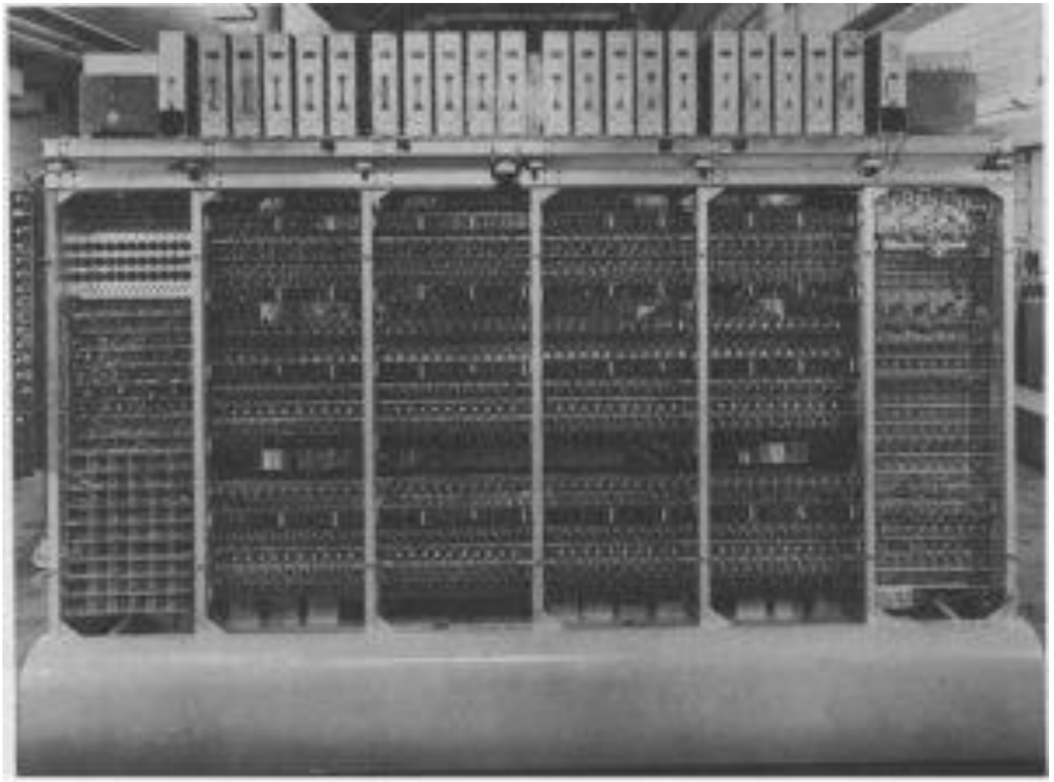
When? 1953

What? First ever numerical simulation

Problem : N coupled anharmonic resonators

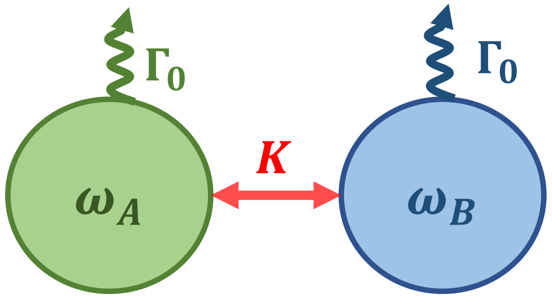
$$\ddot{x}_i = (x_{i+1} + x_{i-1} - 2\,x_i) + \alpha\left[(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2\right]$$

How? With this big thing →



Conclusion : For the last 70 years, it has been easier to run numerical experiments than to set up a proper physical experiment.

Motivation : Non-hermitian physics concepts



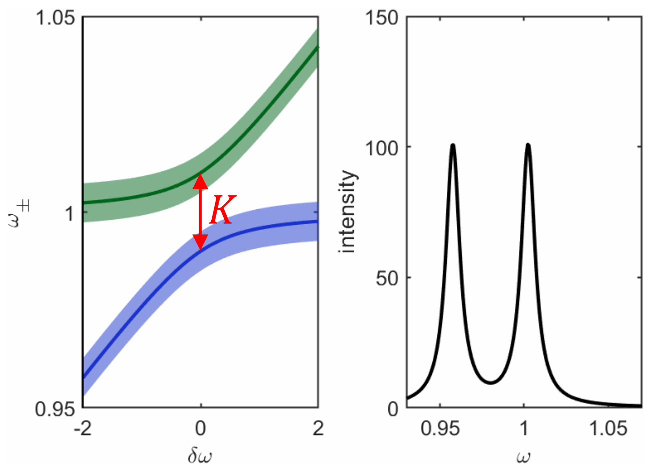
Rate equations for the cavity amplitudes:

$$i\begin{pmatrix}\dot{a}_A\\ \dot{a}_B\end{pmatrix} = \begin{pmatrix}\omega_A - i\Gamma_0 & K\\ K & \omega_B - i\Gamma_0\end{pmatrix}\begin{pmatrix}a_A\\ a_B\end{pmatrix}$$

Resulting eigenvalues:

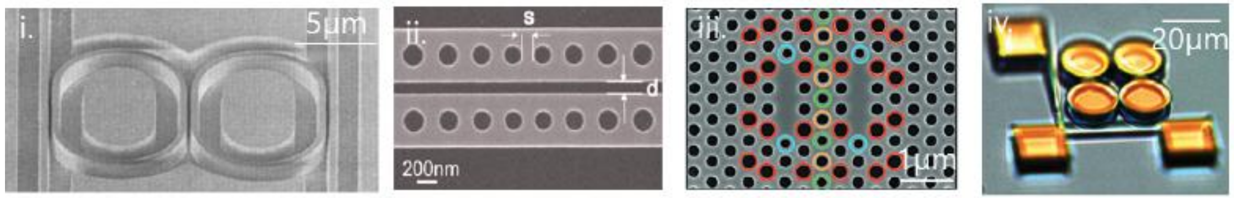
$$\Lambda_\pm = \underbrace{\omega_0 - i\Gamma_0}_{average} \pm \underbrace{\sqrt{\delta\omega^2 + K^2}}_{splitting}$$

Real splitting



- Energy levels repulsion (avoided crossing)
- Identical decay rates

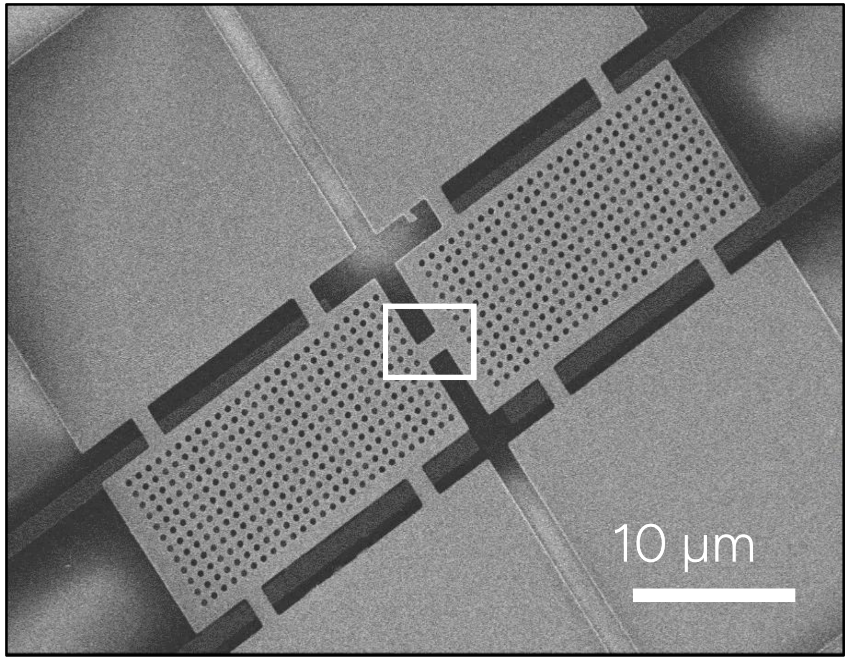
Achievable with passive components

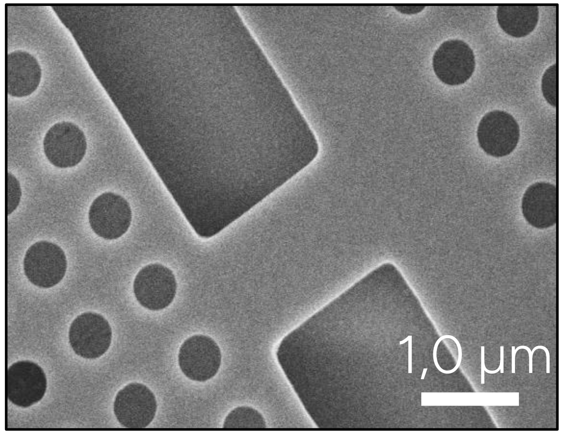


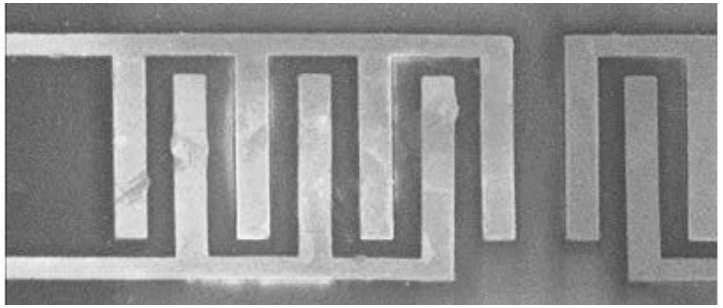
Hryniewiez *et al.*, IEEE PTL 12-3 (2000) — Deotare *et al.*, APL 95, 031102 (2009) — Hamel *et al.*, Nat. Phot. 9, 311 (2015) — Shim *et al.*, Science 316-5821 (2007)

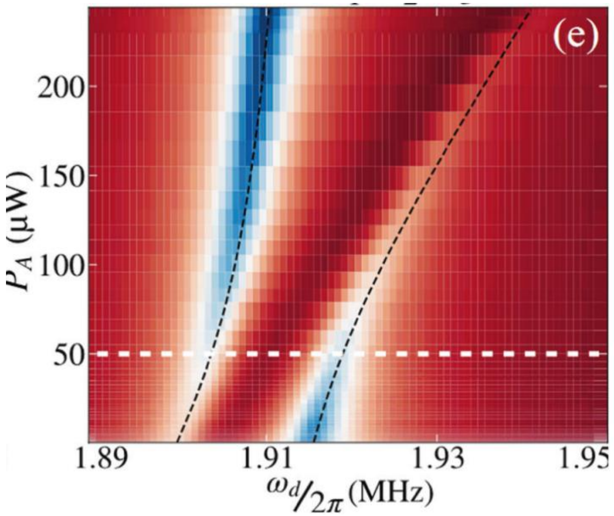
Nano-electromechanics

Coupled membranes

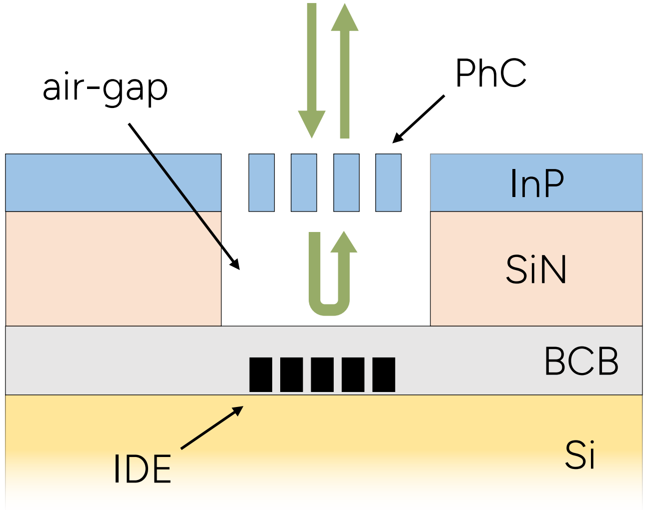


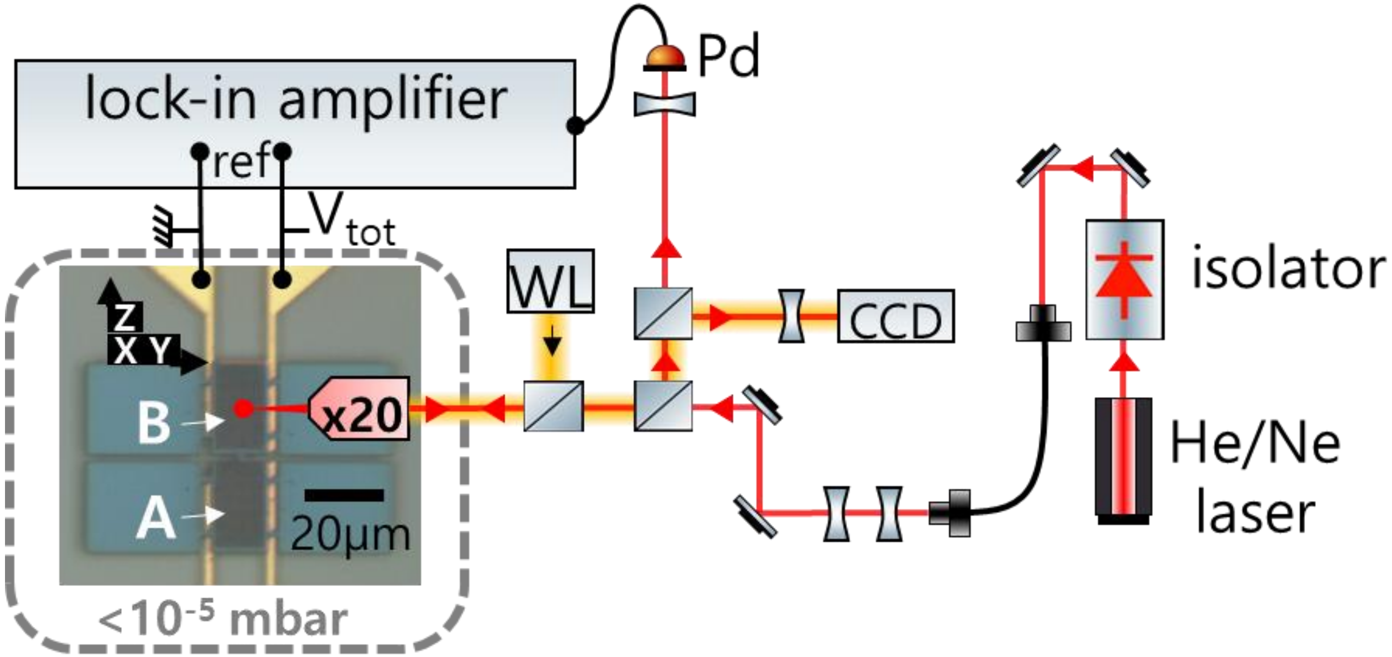






- Pair of 20×10 µm^2 suspended membranes
- Enhanced mechanical Q factor
- Coupling beam
- Independent interdigitated electrodes for integrated actuation
- Optomechanical access for the readout



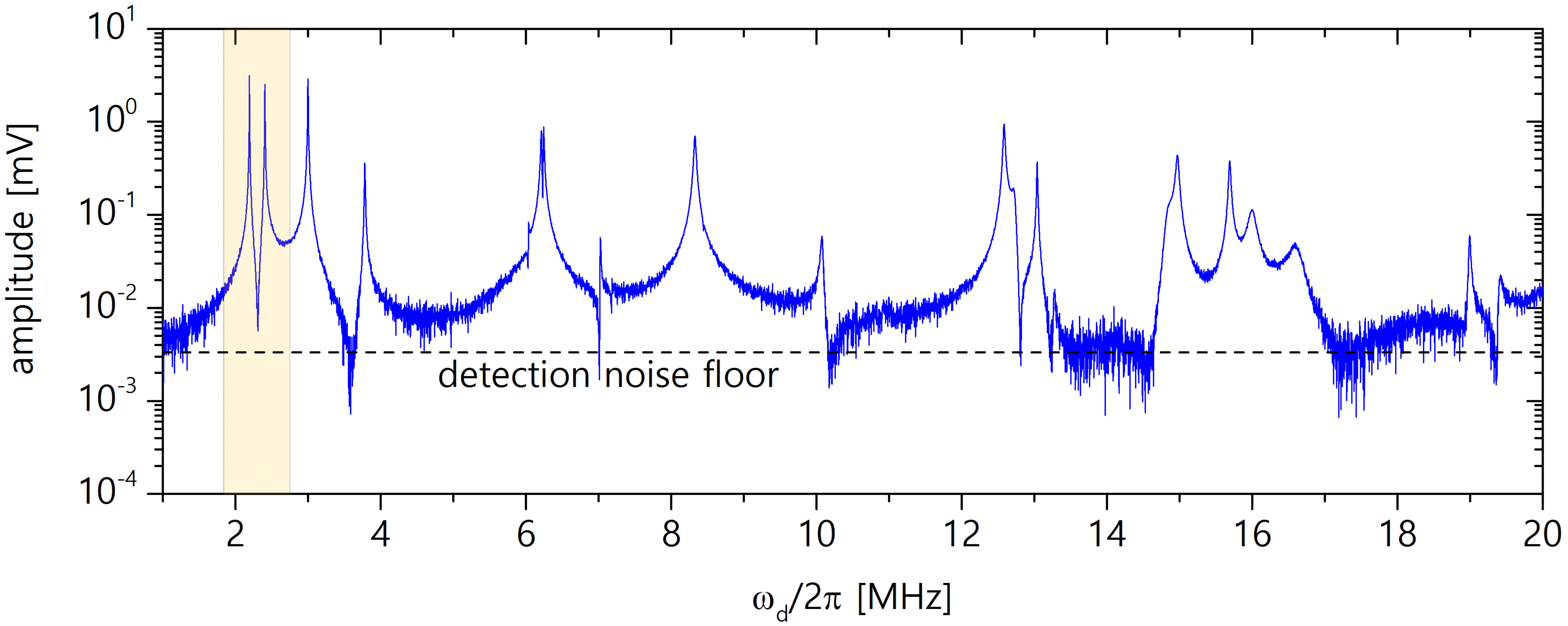


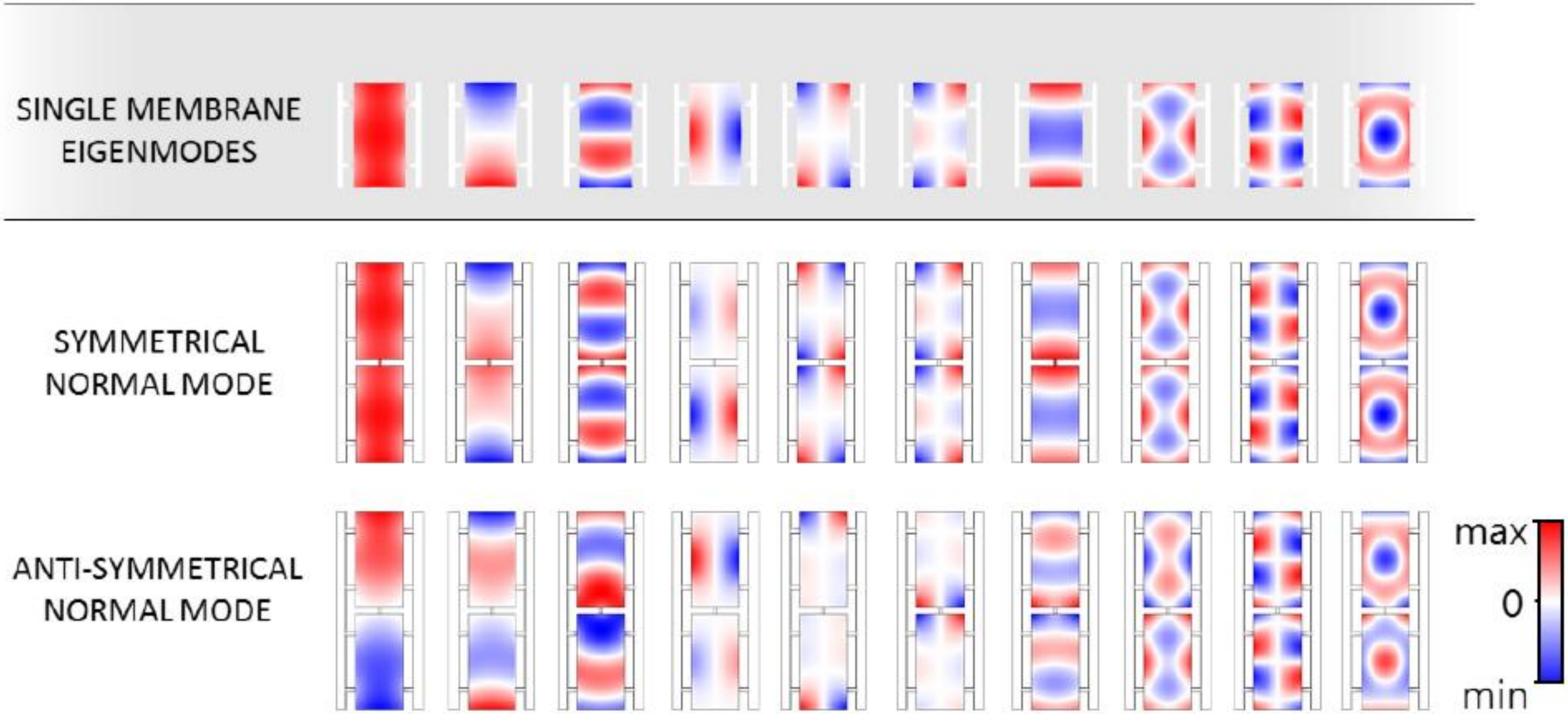
- Apply $V_{tot} = V_{dc} + V_{ac}\cos(\omega_d t)$
- demodulation quadratures at ω_d

Amplitude R_A, R_B

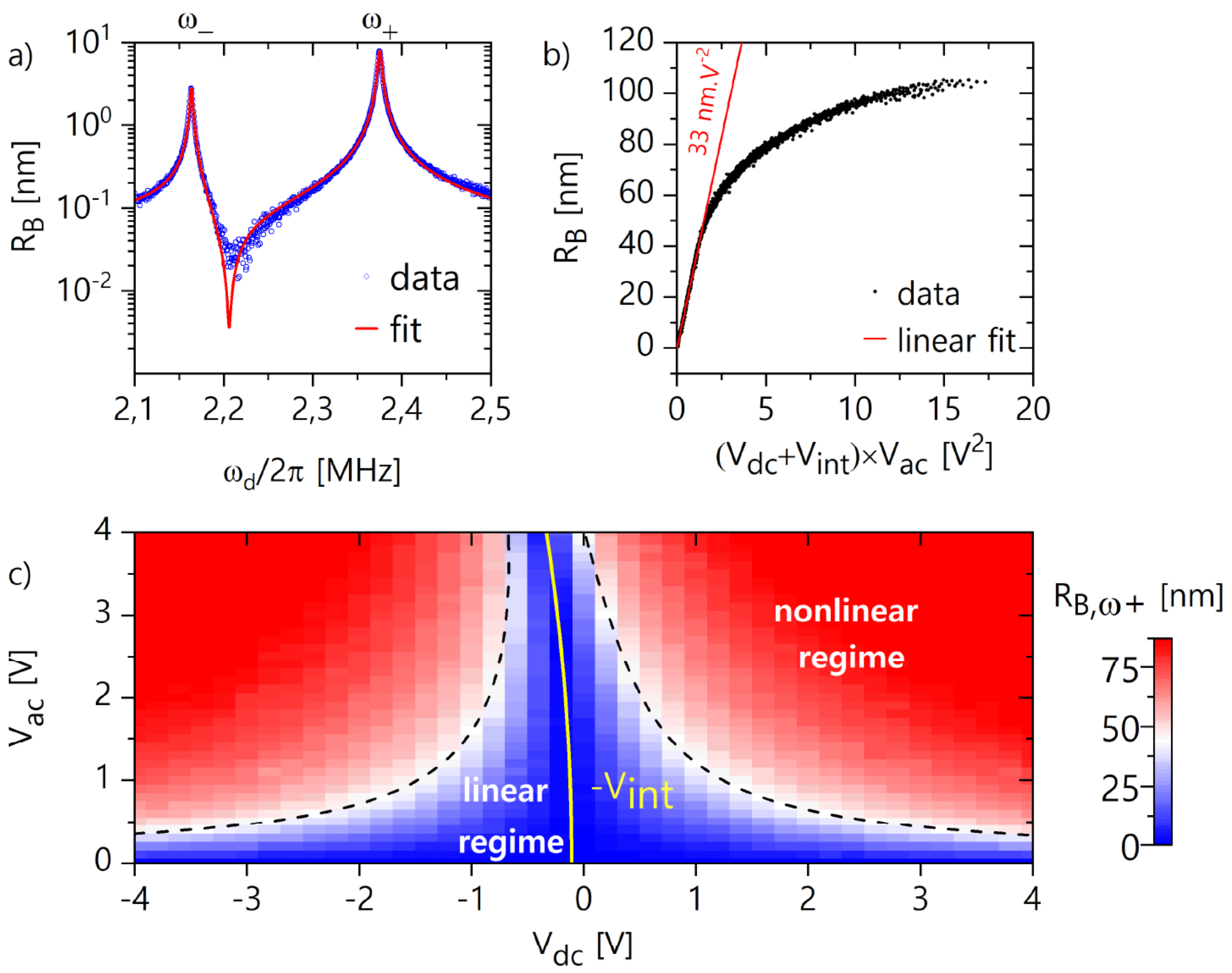
Phase $\theta_A,\ \theta_B$

Mechanical spectrum




SINGLE MEMBRANE EIGENMODES
SYMMETRICAL NORMAL MODE
ANTI-SYMMETRICAL NORMAL MODE
max
0
min

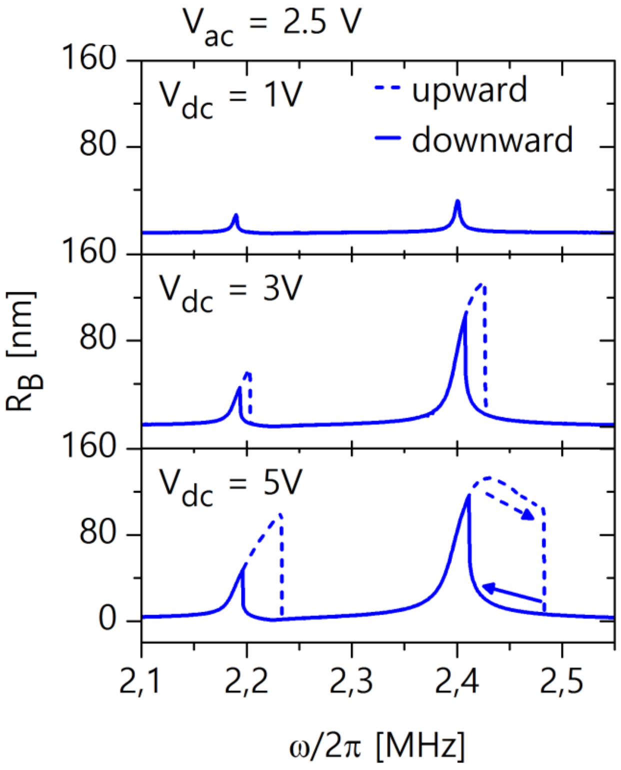
Linear regime

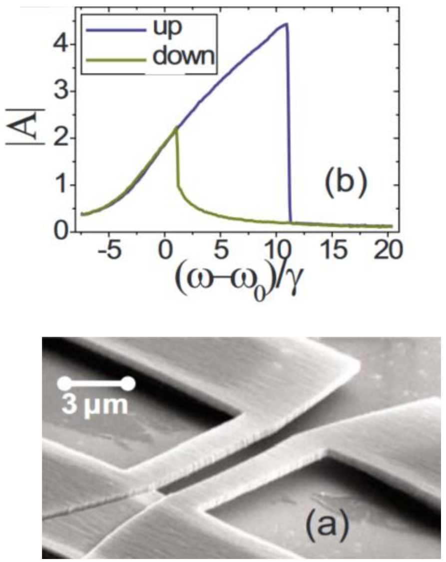


$$F = -\frac{1}{2}\frac{dC}{dx}V_{tot}^2$$

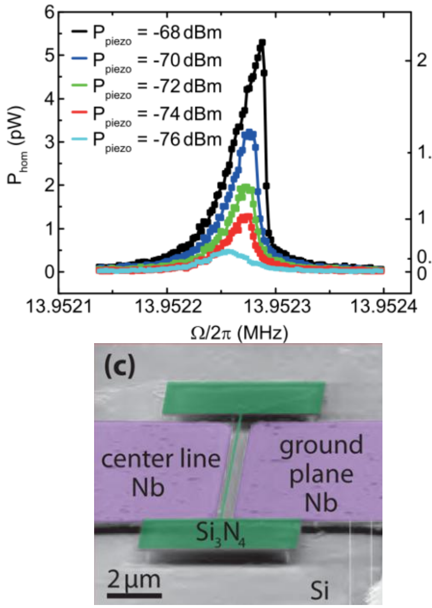
$$\approx \frac{dC}{dx}V_{dc}V_{ac} \times \cos(\omega_d t)$$

$$\begin{cases} \ddot{x}_A + \Gamma_A \dot{x}_A + \omega_A^2 x_A + \boldsymbol{\beta} x_A^3 + G x_B = 0 \\ \ddot{x}_B + \Gamma_B \dot{x}_B + \omega_B^2 x_B + \boldsymbol{\beta} x_B^3 + G x_A = F_B \cos(\omega_d t) \end{cases}$$

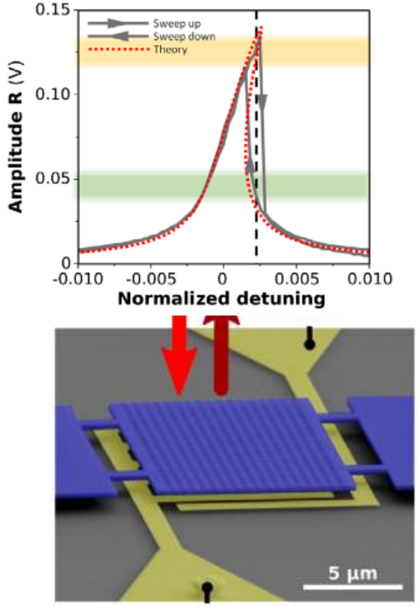




Karabalin et al., PRB (2009)



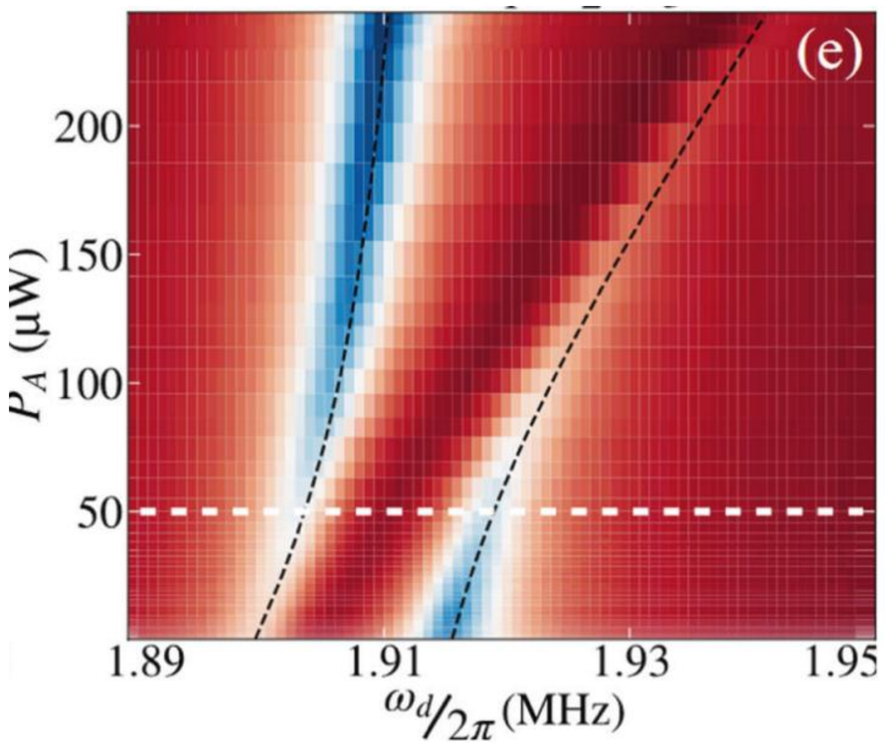
Pernpeintner et al., APL (2014)

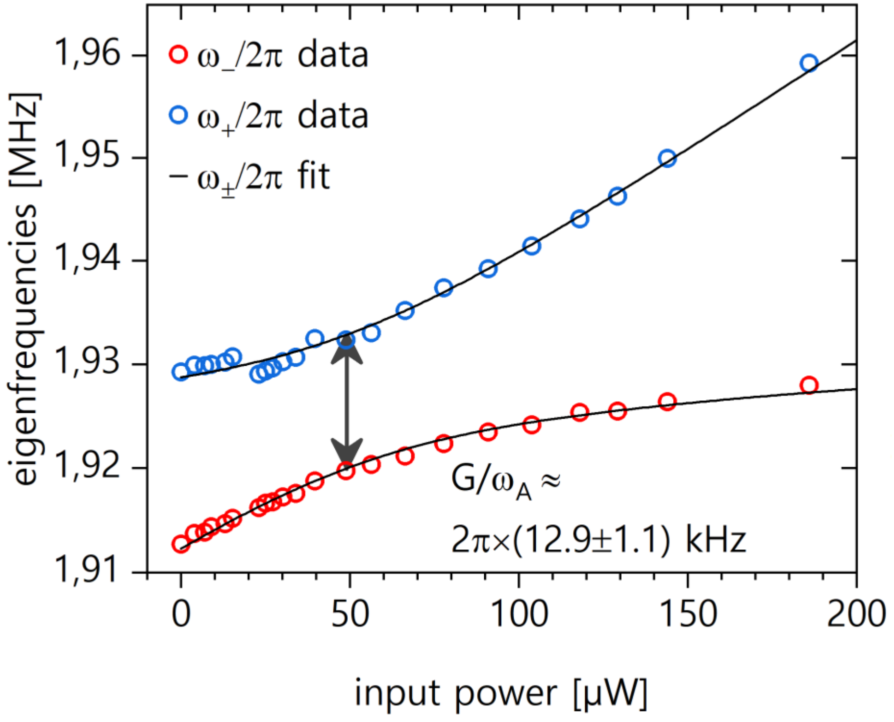


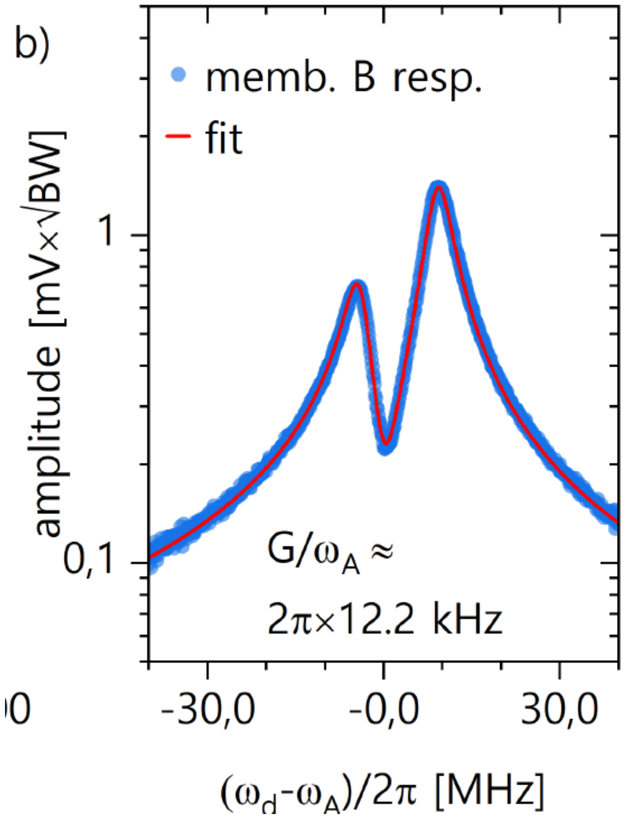
Chowdhury et al., PRL (2017)

…

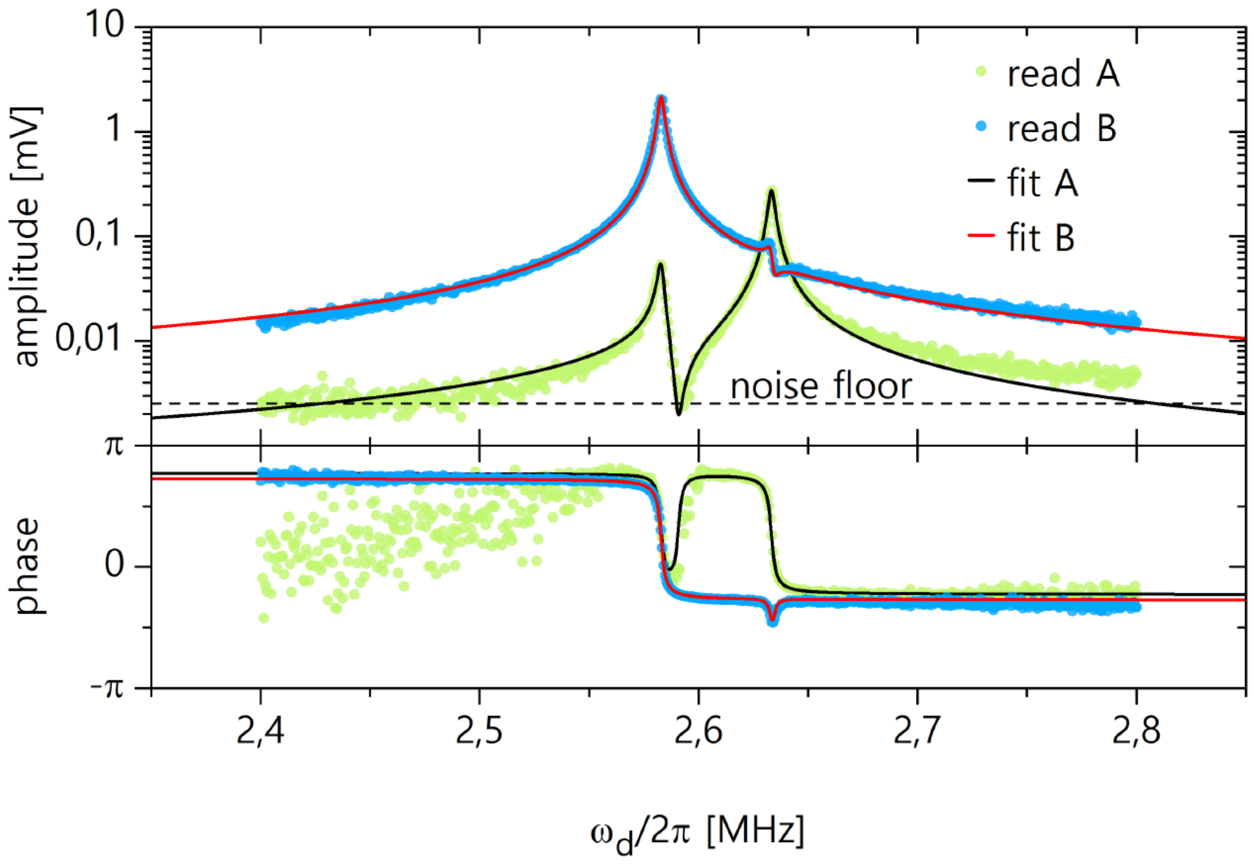
Coupling characterization



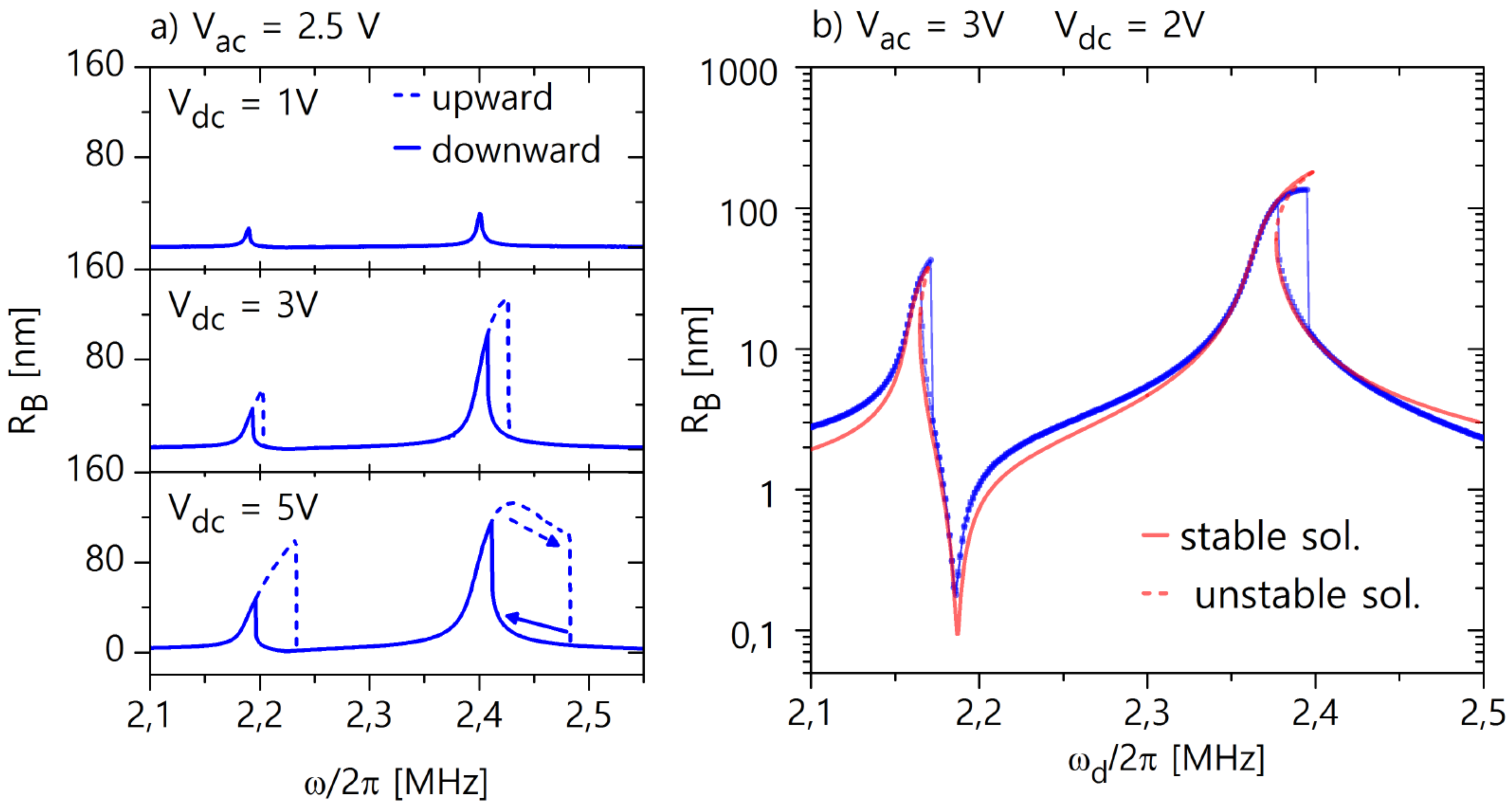




$$\begin{cases} \underline{r_A} = \dfrac{g\tilde{f}_B e^{i\phi} + \left[2\delta - i\gamma_B\right]\tilde{f}_A}{g^2 - \left[2(\delta - \Delta\omega) - i\gamma_A\right]\left[2\delta - i\gamma_B\right]} \\ \\ \underline{r_B} = \dfrac{g\tilde{f}_A + \left[2(\delta - \Delta\omega) - i\gamma_A\right]\tilde{f}_B e^{i\phi}}{g^2 - \left[2(\delta - \Delta\omega) - i\gamma_A\right]\left[2\delta - i\gamma_B\right]} \end{cases}$$



Duffing regime

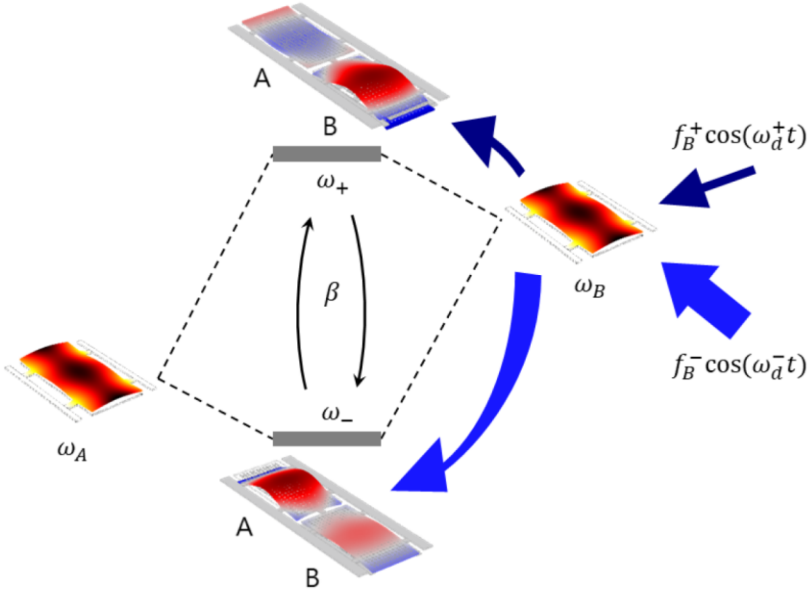


$$\dot{r}_A = \frac{-\gamma_A}{2} r_A + \frac{g}{2} r_B \sin(\vartheta_A - \vartheta_B)$$

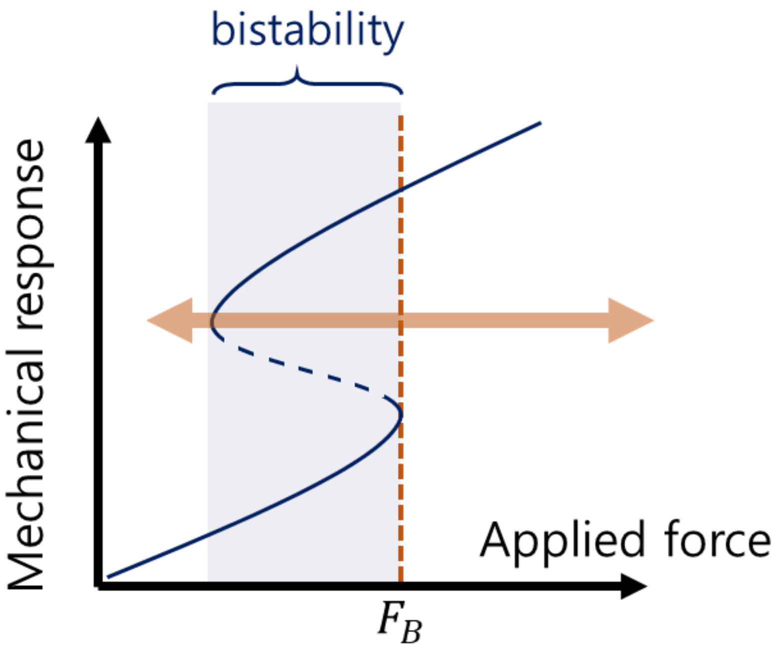
$$\dot{r}_B = \frac{-\gamma_B}{2} r_B - \frac{g}{2} r_A \sin(\vartheta_A - \vartheta_B) + \frac{\tilde{f}_B}{2} \sin(\vartheta_B)$$

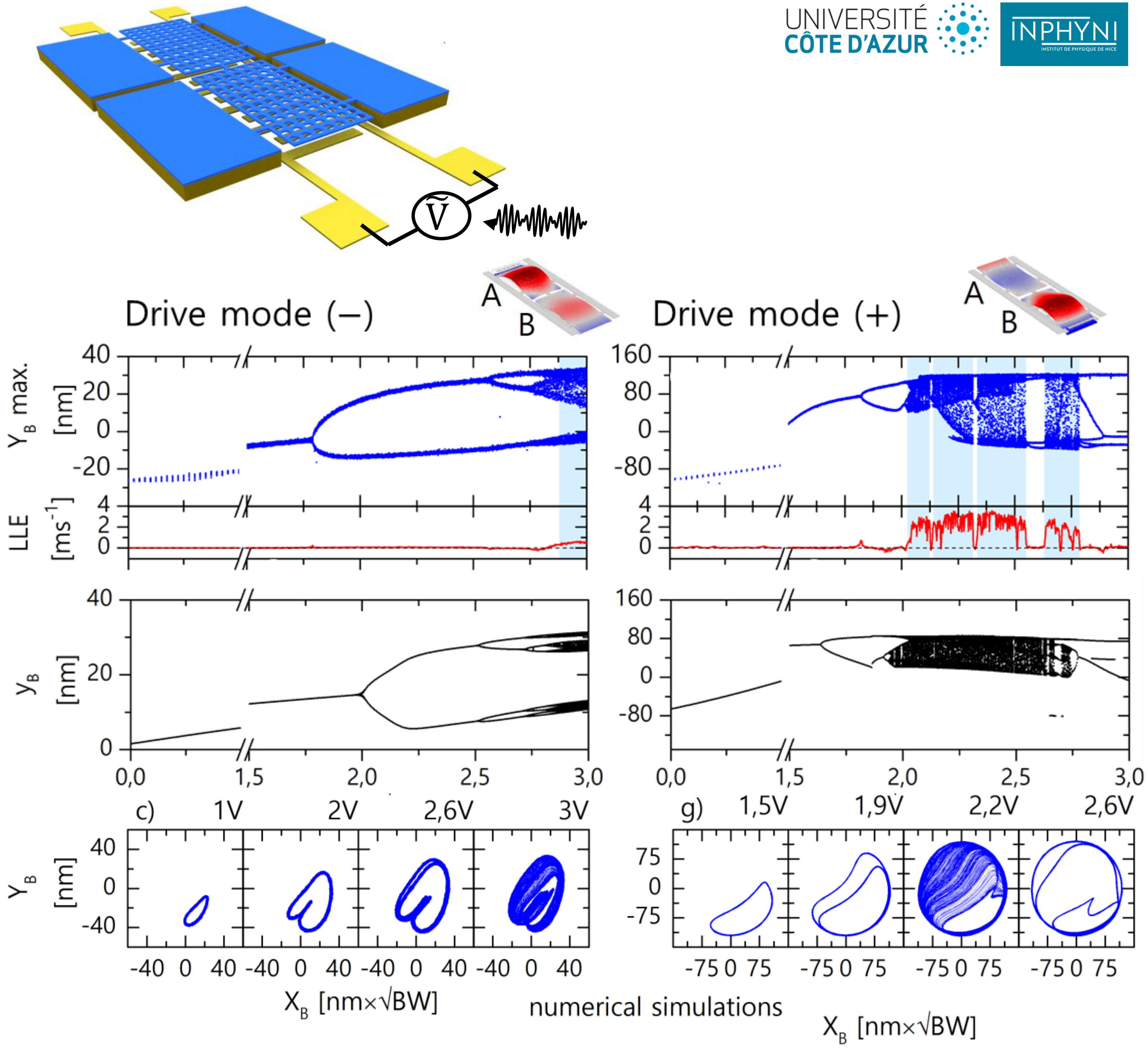
$$r_A \dot{\vartheta}_A = \frac{-r_A}{2} \left[2(\delta - \Delta\omega) + \frac{3}{4} \tilde{\beta} r_A^2 \right] + \frac{g}{2} r_B \cos(\vartheta_A - \vartheta_B$$

$$r_B \dot{\vartheta}_B = \frac{-r_B}{2} \left[2\delta + \frac{3}{4} \tilde{\beta} r_B^2 \right] + \frac{g}{2} r_A \cos(\vartheta_A - \vartheta_B) + \frac{\tilde{f}_B}{2} \cos(\vartheta_B)$$

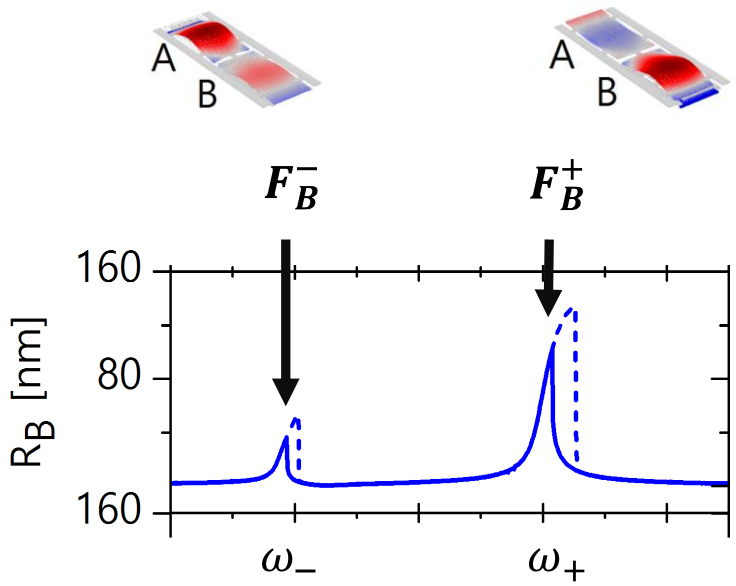


Chaos and synchronization





Bichromatic driving



Bichromatic driving scheme

→ Chaos on both mode ?

$$\begin{cases} \ddot{x}_A + \Gamma_A \dot{x}_A + \omega_A^2 x_A + \beta x_A^3 + G x_B = 0 \\ \ddot{x}_B + \Gamma_B \dot{x}_B + \omega_B^2 x_B + \beta x_B^3 + G x_A = \boldsymbol{F_B^-}(\mathrm{t})\cos(\boldsymbol{\omega_d^-} t) \\ \qquad + \boldsymbol{F_B^+}(t)\cos(\boldsymbol{\omega_d^+} t) \end{cases}$$

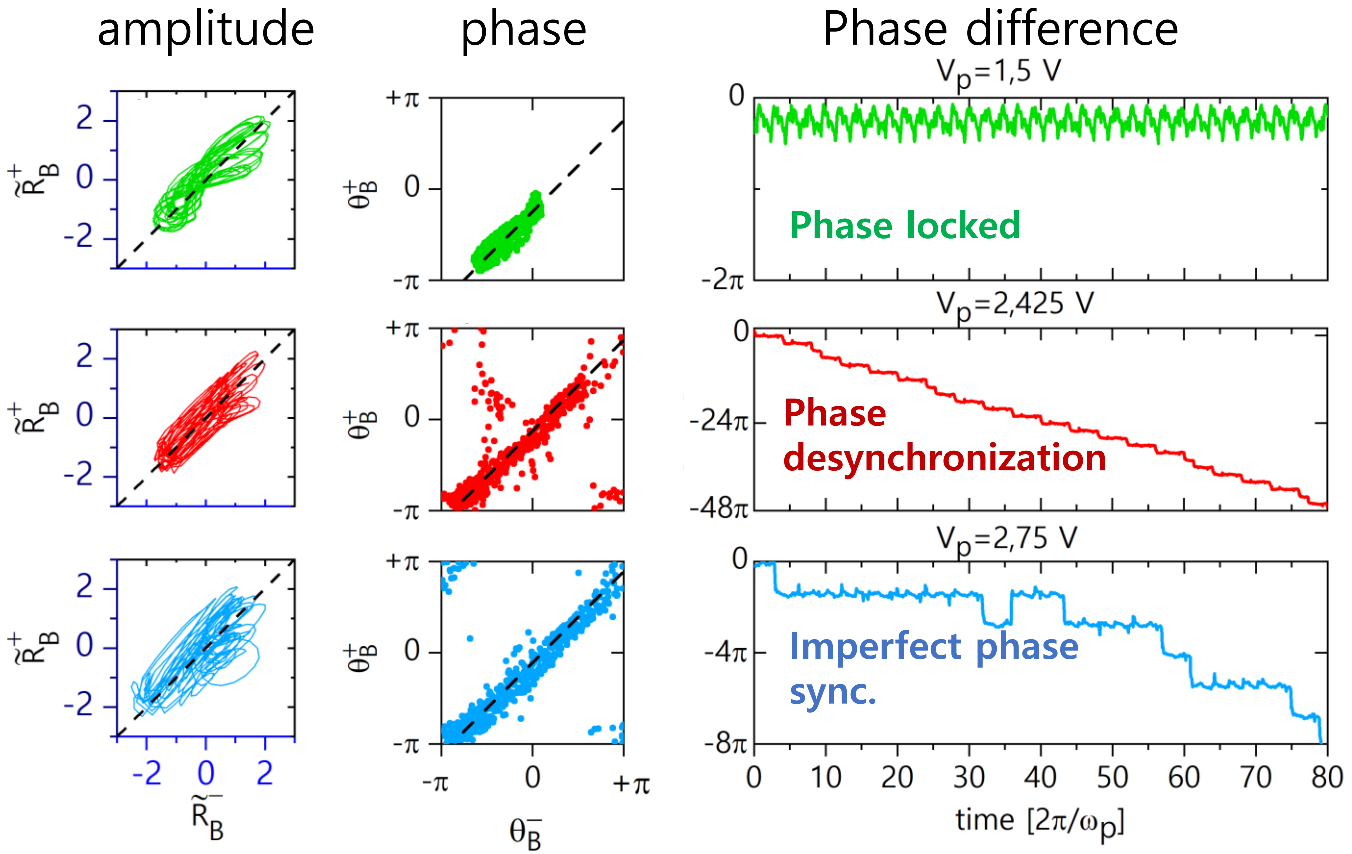
Both modes are **driven** and **probed** through membrane B

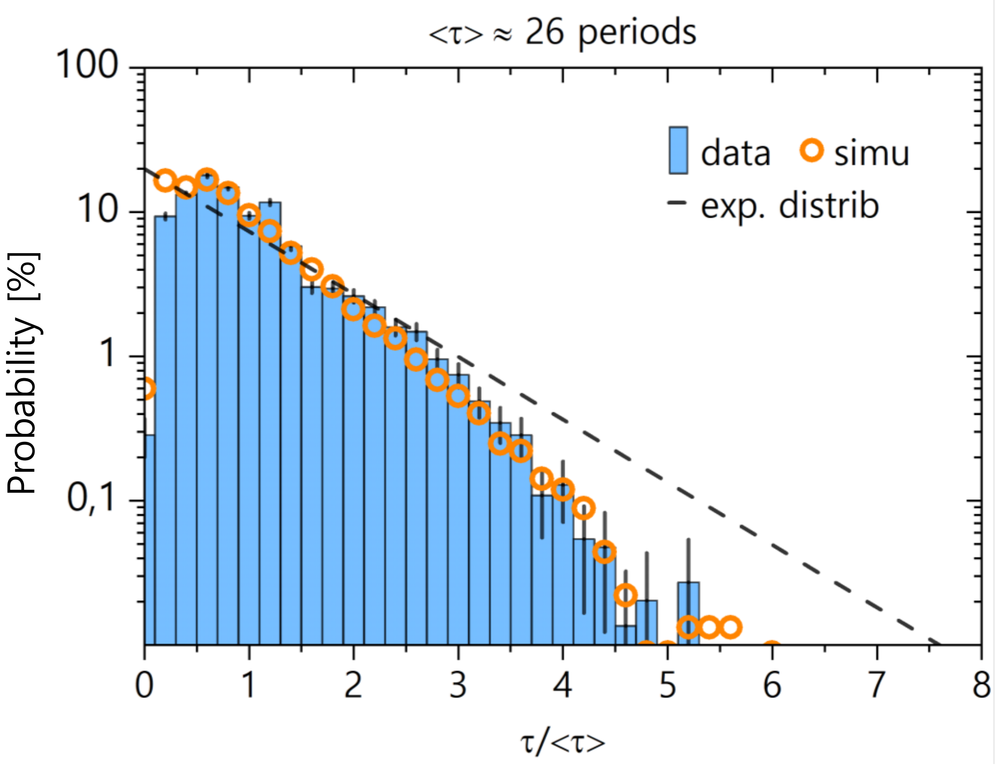
The displacement of membrane B is demodulated:

- **at $\boldsymbol{\omega_d^-}$** → amplitude/phase of mode (-) (R_B^-, θ_B^-)
- **at $\boldsymbol{\omega_d^+}$** → amplitude/phase of mode (+) (R_B^+, θ_B^+)

Imperfect phase locking

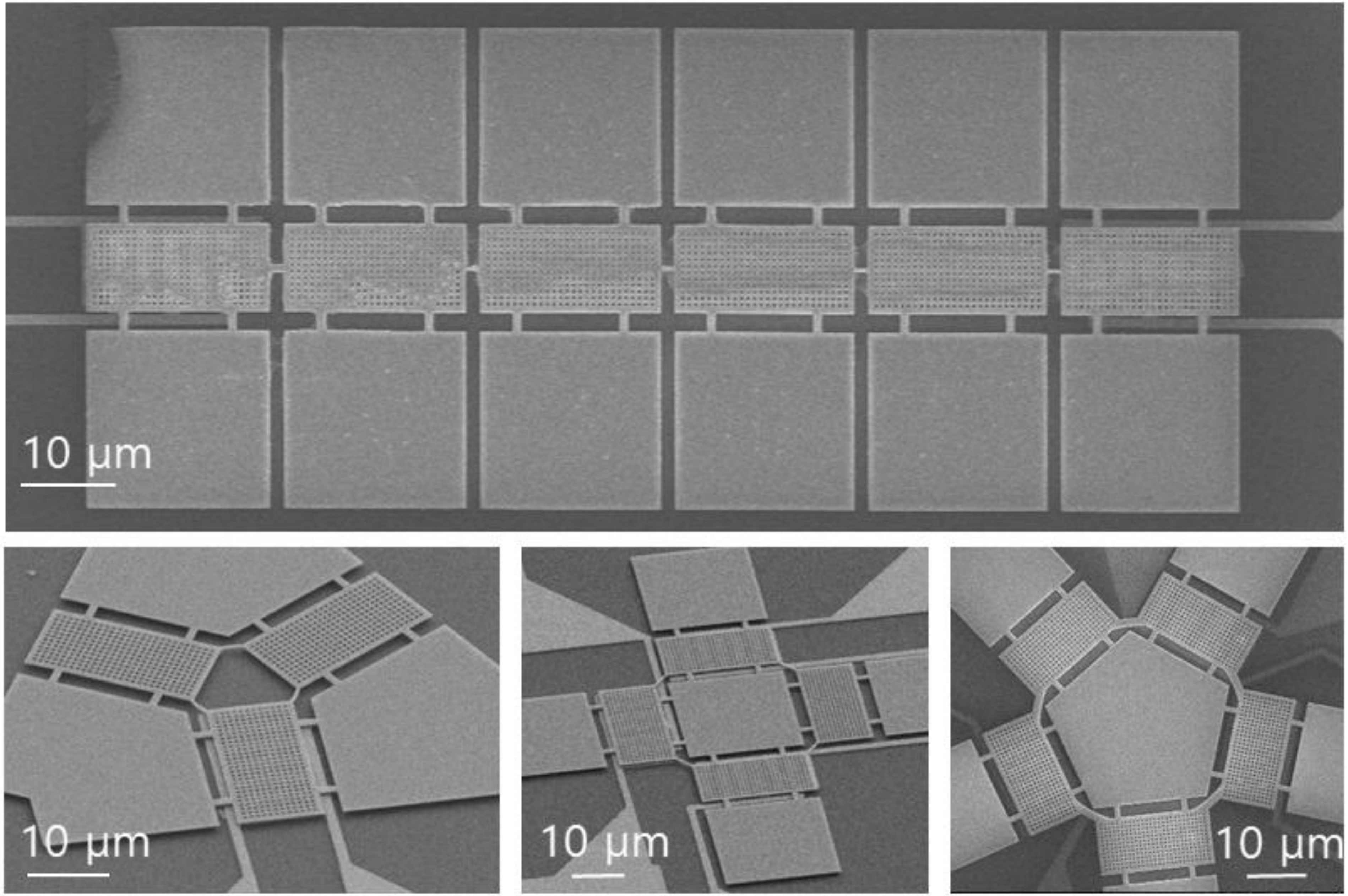
Relative dynamics





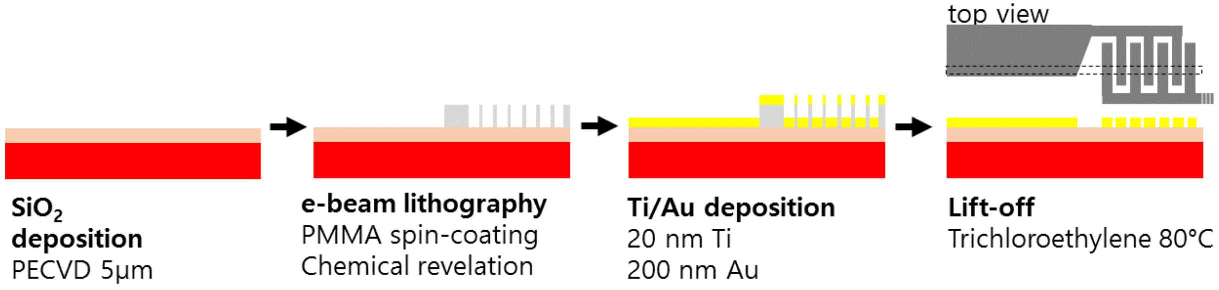
Non exponential distribution

→ Attest the **deterministic** nature of this process

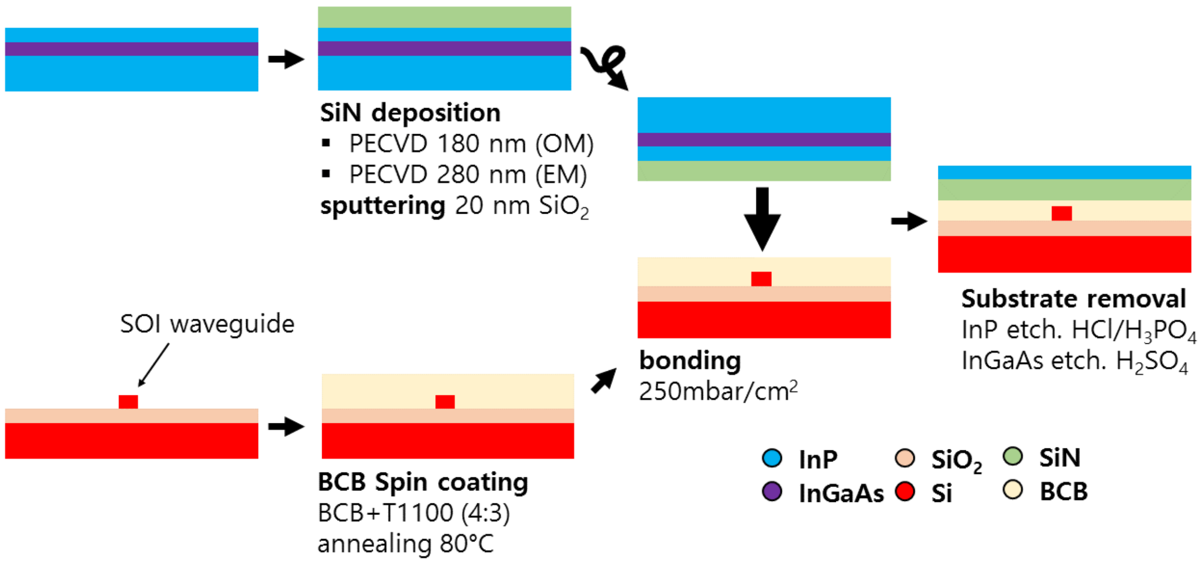

10 µm
10 µm
10 µm
10 µm

Nanofab

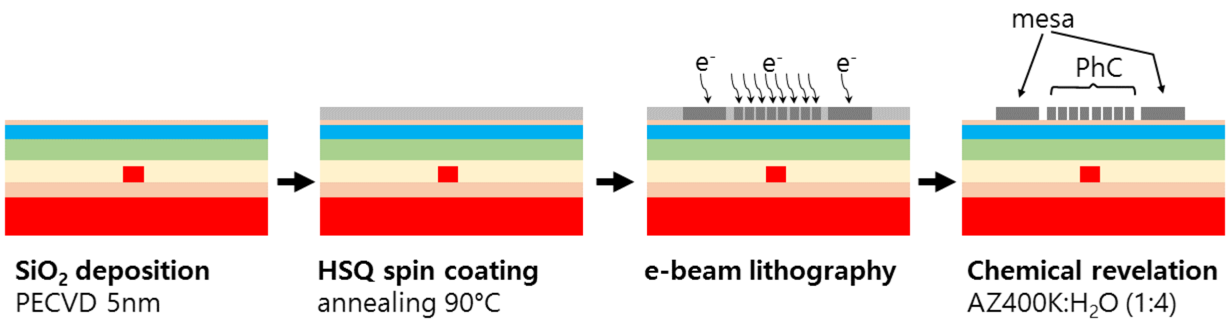
A) IDEs integration



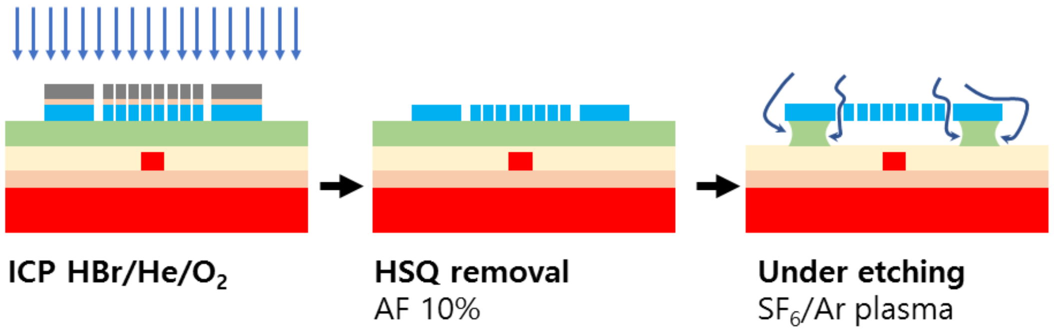
B) Heterogeneous BCB bonding



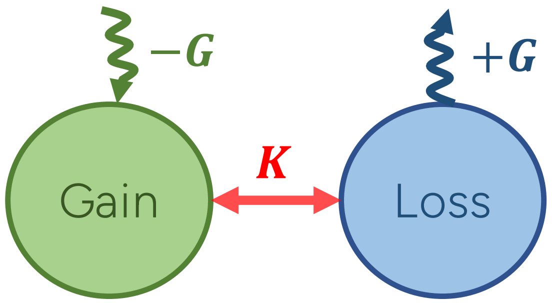
C) e-beam lithography



D) III-V etching + under-etching



Motivation : Non-hermitian physics concepts



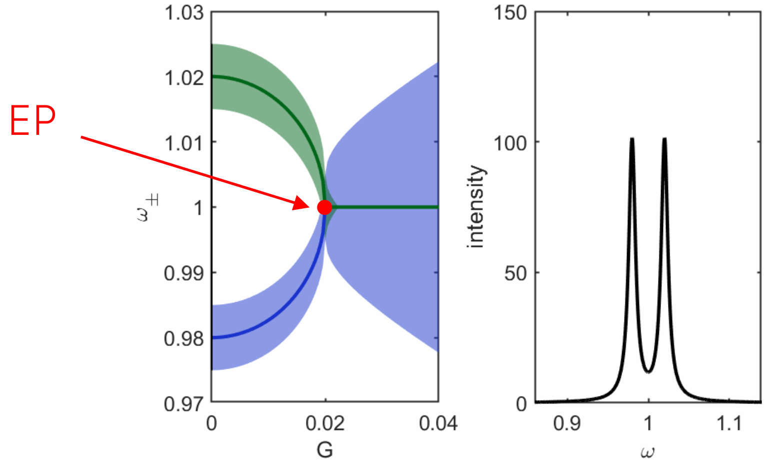
Rate equations for the cavity amplitudes:

$$i\begin{pmatrix}\dot{a}_A\\ \dot{a}_B\end{pmatrix} = \begin{pmatrix}\omega_0 + iG & K\\ K & \omega_0 - iG\end{pmatrix}\begin{pmatrix}a_A\\ a_B\end{pmatrix}$$

Resulting eigenvalues:

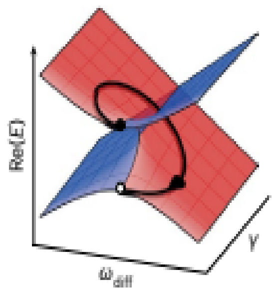
$$\Lambda_{\pm} = \underbrace{\omega_0 - i\Gamma_0}_{average} \pm \underbrace{\sqrt{K^2 - G^2}}_{splitting}$$

Complex splitting

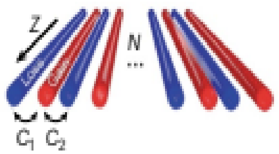


- Loss-splitting
- Identical frequencies
- Crossing of an exceptional point (EP) $(\Lambda_- = \Lambda_+)$

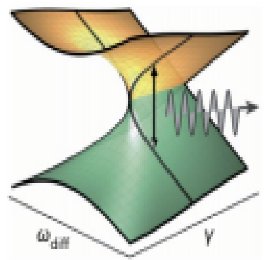
Nonreciprocity



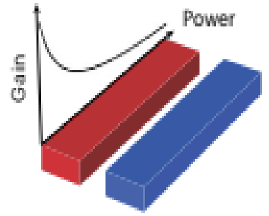
Topology



Sensing



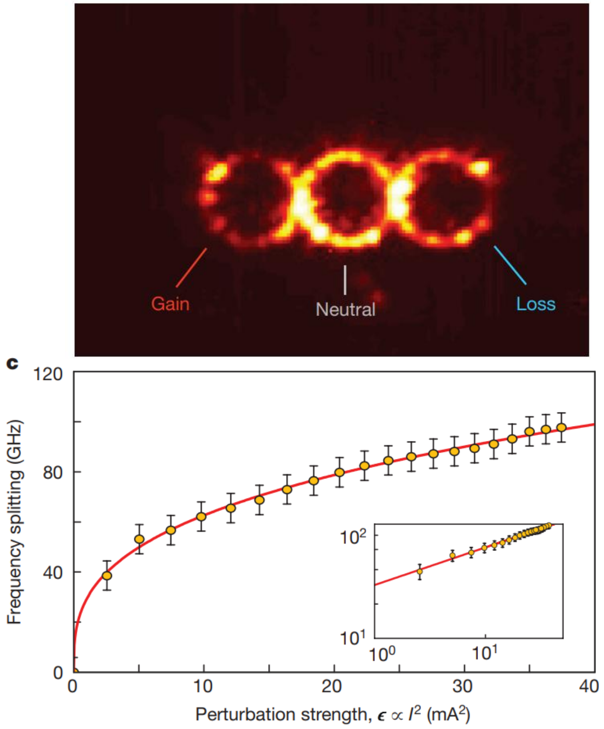
Nonlinear optics



Feng *et al.*, Nat. Photon. 11 752-762 (2017)
Hokmabadi *et al.*, Nature 576, 70–74 (2019)
Li *et al.*, Nature Nanotech. 18 706-720 (2023)

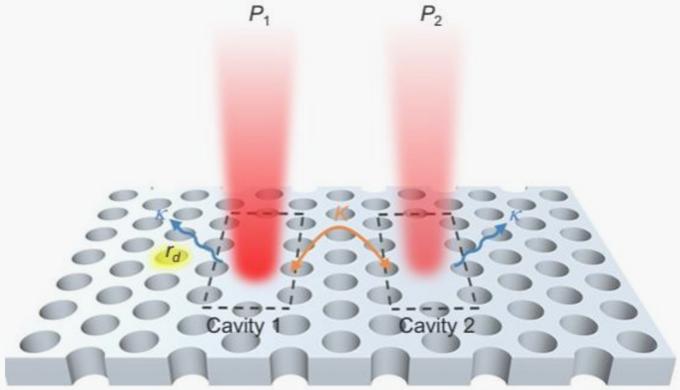
Some experimental demonstrations

Sensing



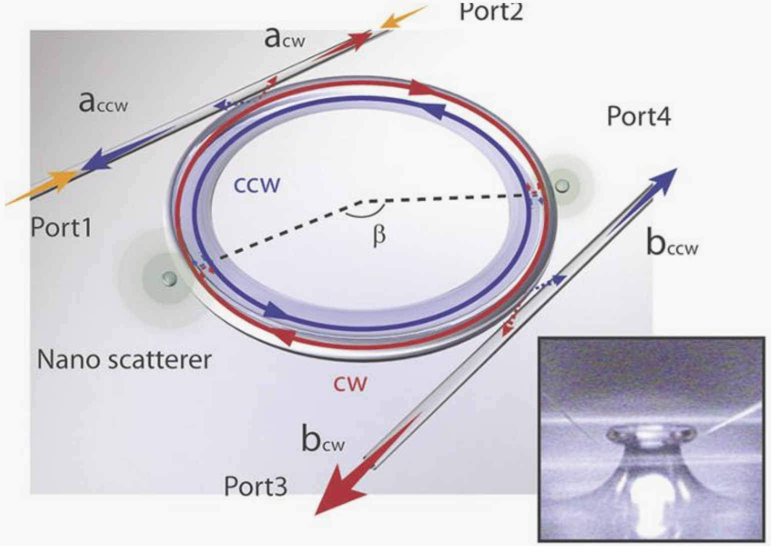
Hodai *et al.*, Nature (2017)

Above-threshold investigation



Ji *et al.*, Nature Com. 14-8304 (2023)

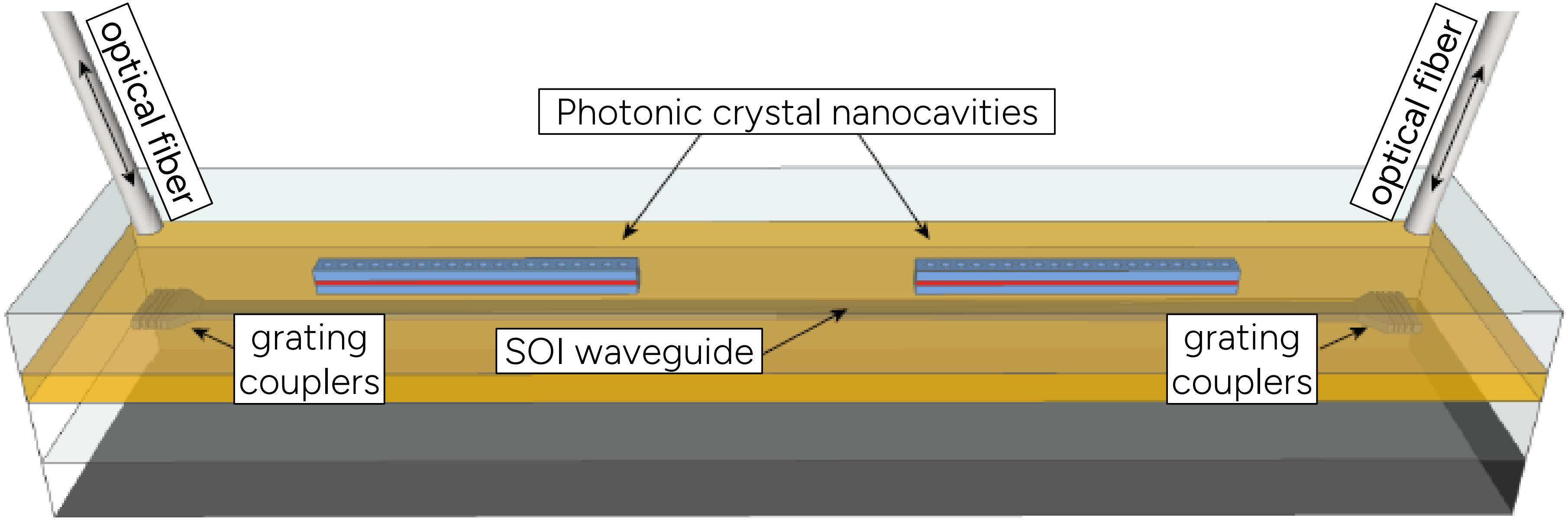
Directional lasing emission

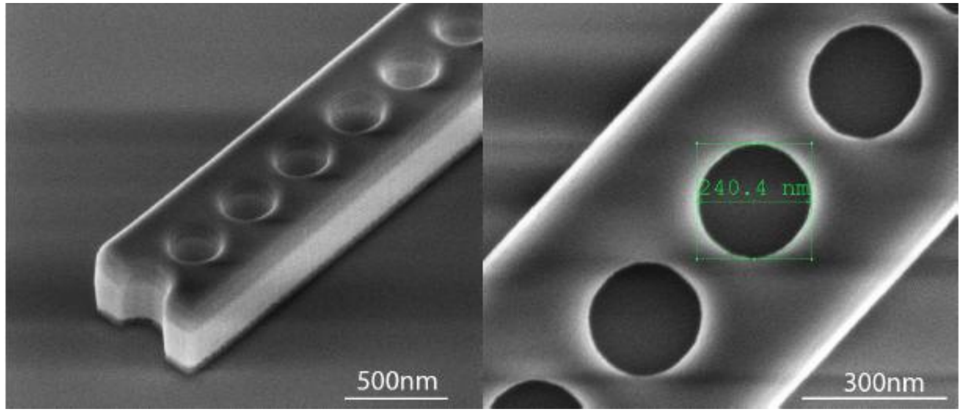


Peng *et al.*, PNAS 113-25 (2016)

Can these functionalities be integrated and made reconfigurable ?

III-V photonic crysal nanolasers



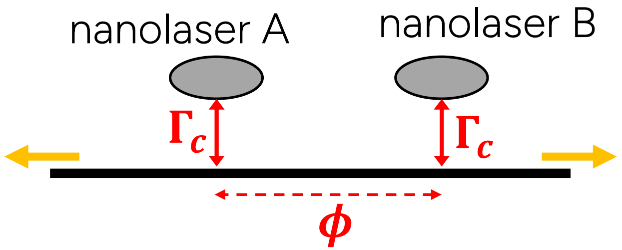


Q. Chateiller, *PhD thesis*, Univ. Paris Saclay (2020)

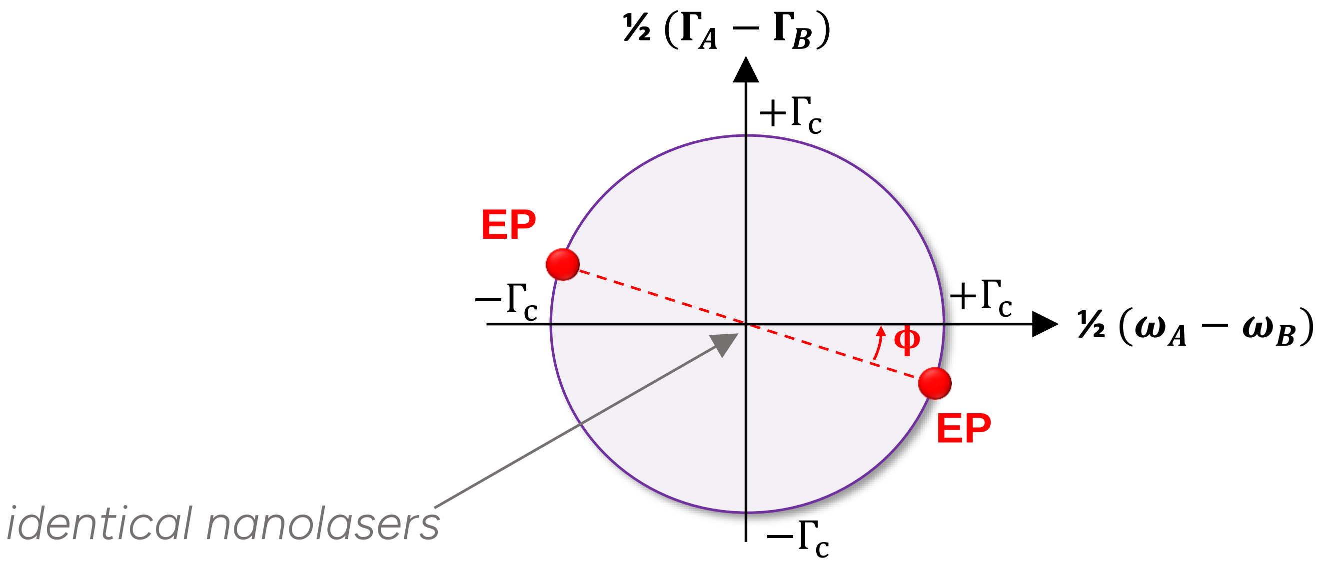
- High Q/V nanocavity
- InGaAsP quantum wells providing gain
- Integrated SOI waveguide
- Separation timescale is below the photon lifetime

Madiot *et al*, Science Advances 10,45 (2024)

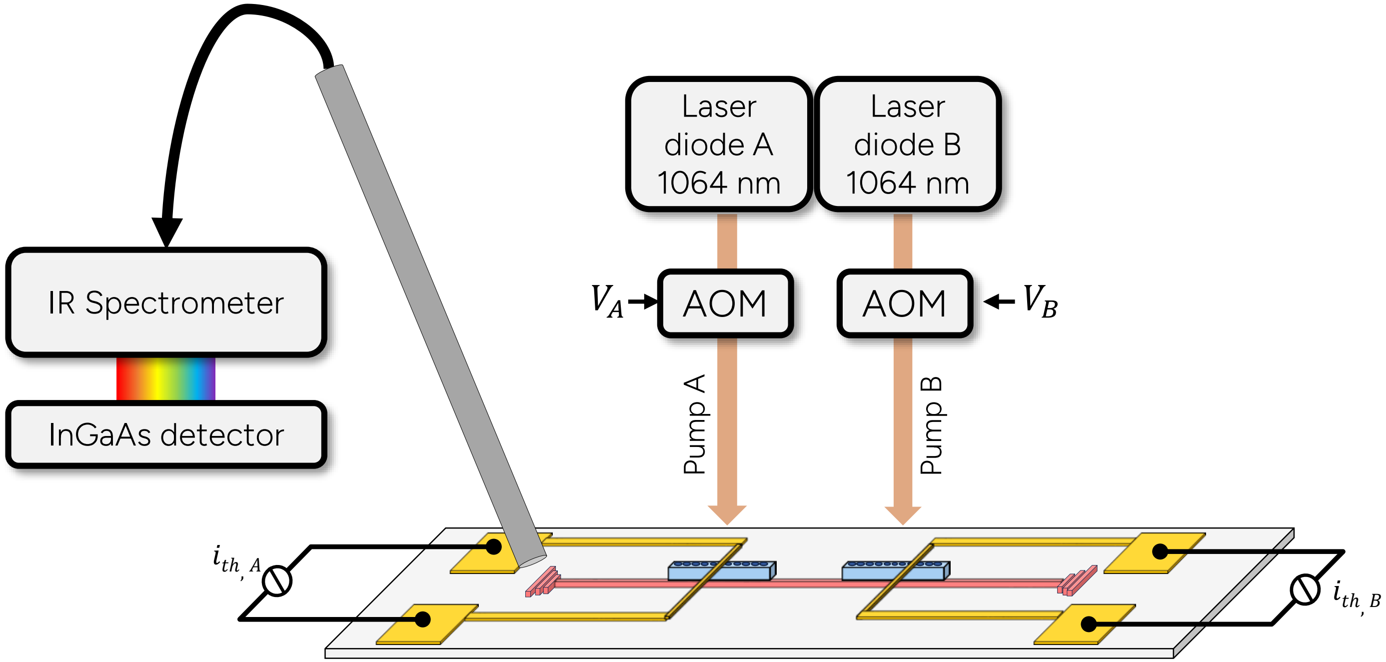
Coupled mode analysis

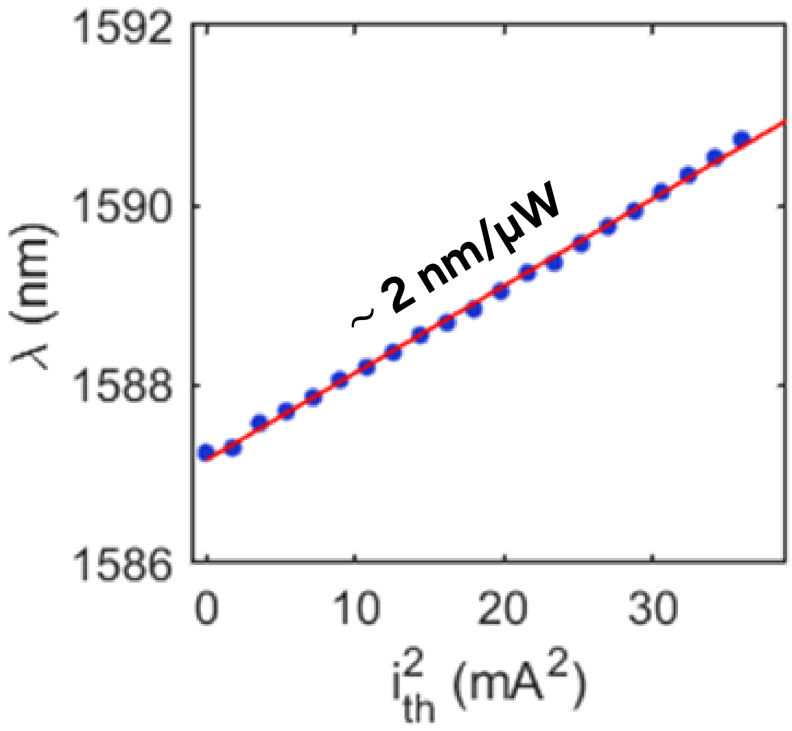


- The coupling rate is complex
- Its dispersive ($\mathbb{R}$) and dissipative ($i\mathbb{R}$) contributions can be tuned using $\boldsymbol{\Gamma_c}$ and $\boldsymbol{\phi}$
- Exceptional Points (EP) can be found by properly tuning the nanolasers frequencies and gain.

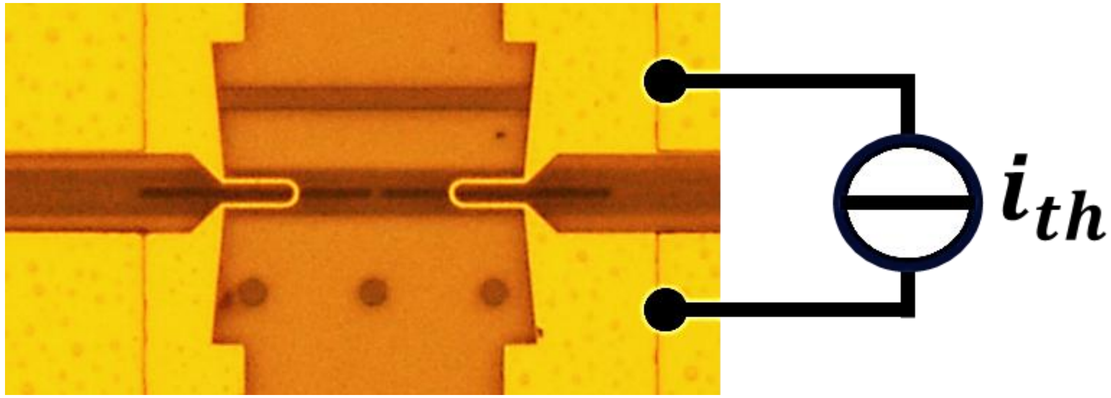


Tuning of the emission wavelength





- Integrated SOI waveguide
- InGaAsP nanocavities
- Gold thermoresistors



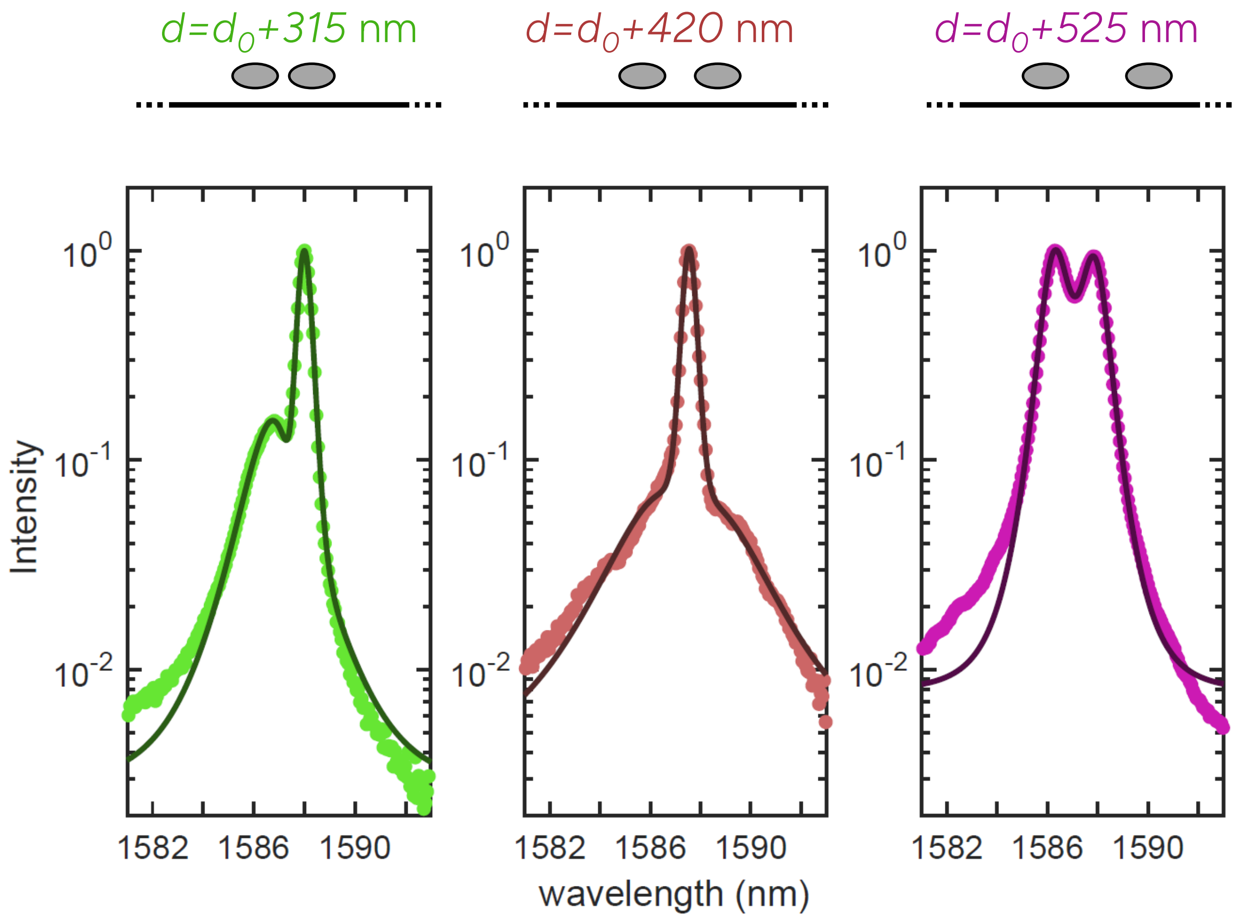
Nano-heaters

→ Thermo-optic tuning of the cavities up to +3 nm

→ *Fine control over the cavity detuning $\delta\omega$*

Extraction of the eigenvalues

After setting $\boldsymbol{\delta\omega = 0}$ & $\boldsymbol{\delta\Gamma = 0}$ *:*



Fit extraction of the eigenvalues:

- Frequencies ω_- and ω_+
- Decay rates Γ_- and Γ_+

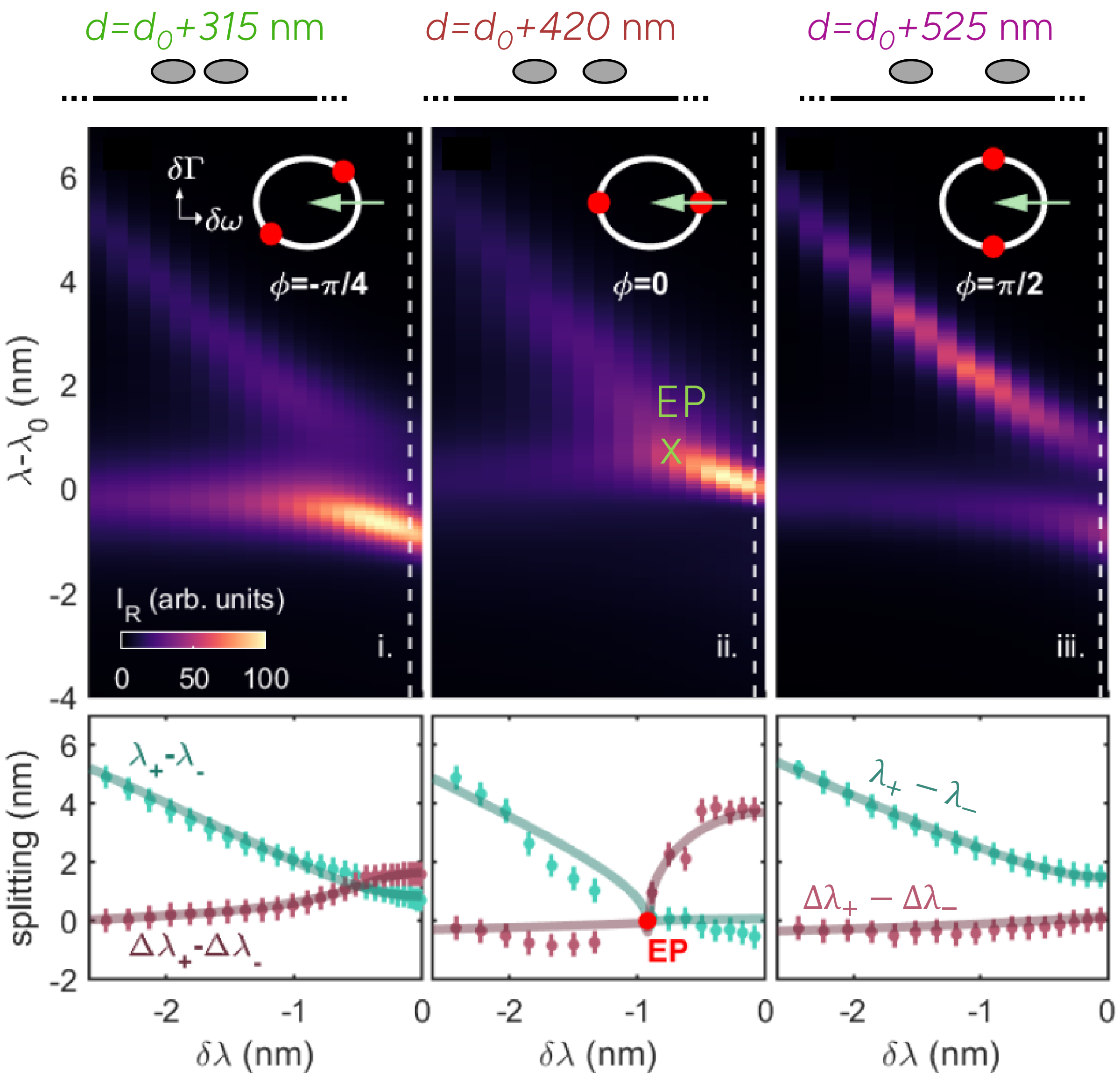
Identification of different coupling regimes:

- *Frequency-splitting*
- *Loss-splitting*
- *« Complex » splitting*

…

→ *Identify exceptional point?*

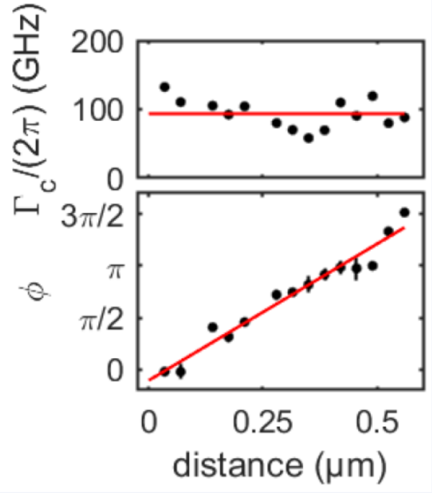
ϕ-dependent coupling regime



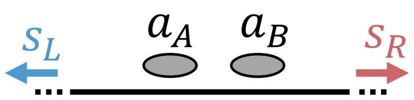
Now varying the detuning $\boldsymbol{\delta\lambda}$

Fitting of the eigenvalues as a function of $\boldsymbol{\delta\lambda}$

- Extraction of the coupling parameters Γ_c, ϕ
- Evidence of an exceptional point when $\phi \approx 0$ and $\delta\omega \approx \Gamma_c$
- Check how they depend on *d* :



Emission directionality

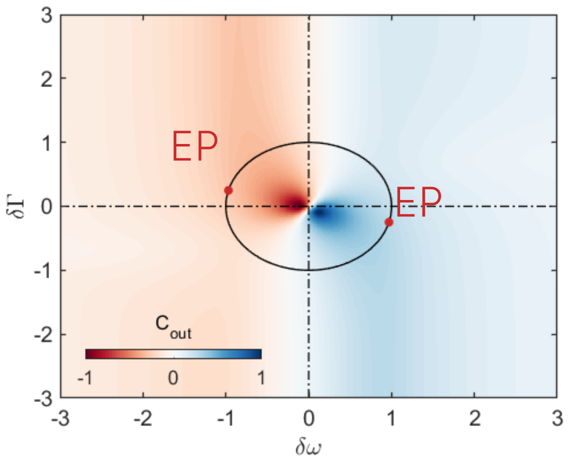


Output wave amplitudes

$$s_L = j\sqrt{\Gamma_c}\left(a_A + e^{-i\phi} a_B\right)$$
$$s_R = j\sqrt{\Gamma_c}\left(e^{-i\phi} a_A + a_B\right)$$

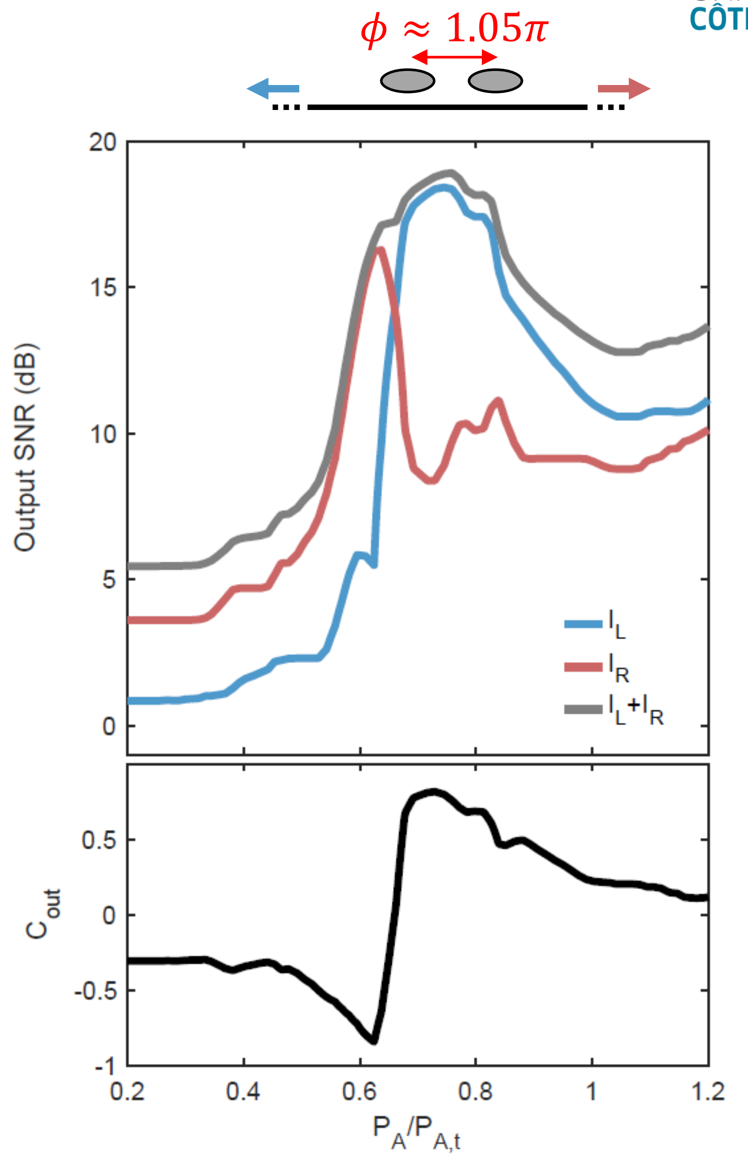
Output intensity contrast

$$C_{out} = \frac{I_L - I_R}{I_L + I_R}, \quad \text{where} \quad I_{L,R} = \left|s_{L,R}\right|^2$$



Directionality flip is possible using the proper trajectory

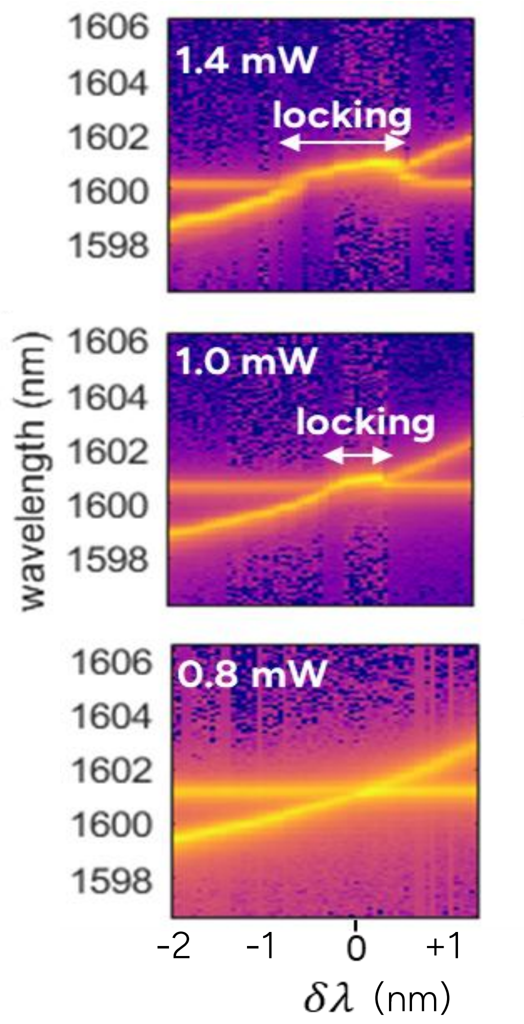
→ Control $\omega_{A,B}$ and $\Gamma_{A,B}$ through the pump powers



- Non identical output intensities
- Inversion of the output contrast

Madiot *et al*, Science Advances 10,45 (2024)

Above-threshold regime

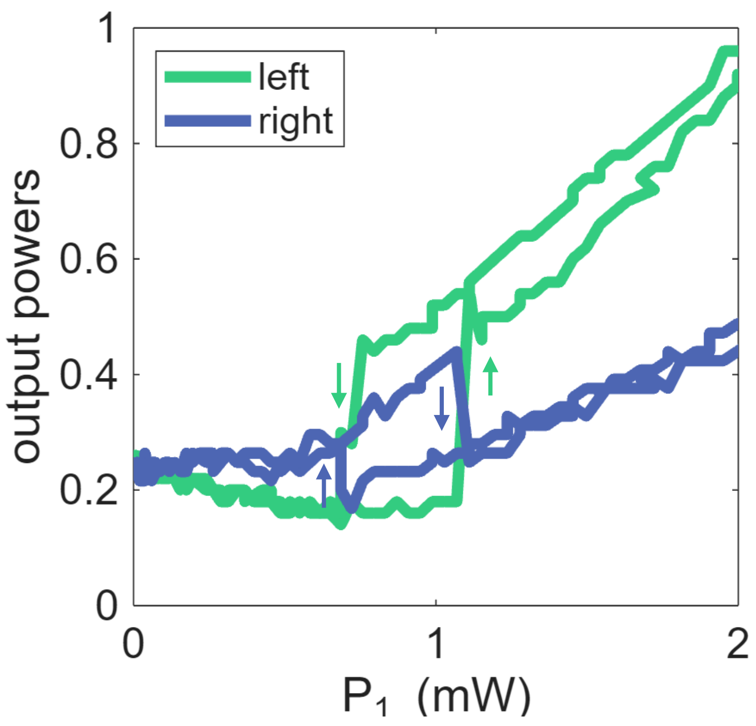


- Phase dynamics

$$\dot{\Psi}_i = \omega_i - \Gamma_c \sqrt{\frac{S_2}{S_1}} \sin(\Psi_1 - \Psi_2 - \phi)$$

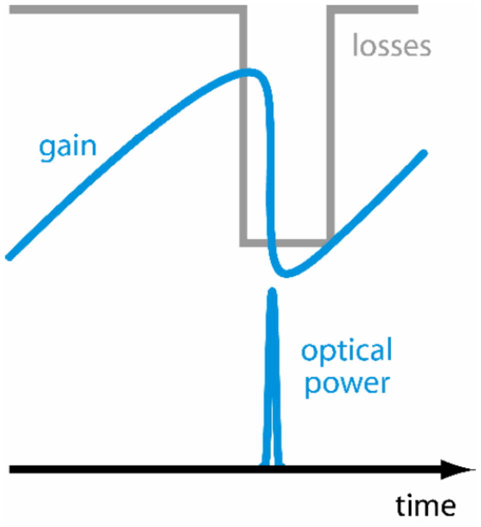
- Temporal delay is negligible

$\tau_\phi = \frac{\phi}{\omega} =$ 10-100 fs $\ll \tau_p$ =10 ps



Q-switched laser

Q-switching: Generation of short pulses by fast tuning of the cavity losses

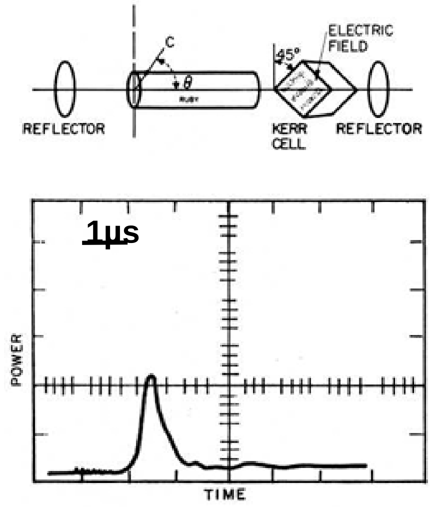


Paschotta, Rüdiger. "Field Guide to Lasers." SPIE Press, 2008

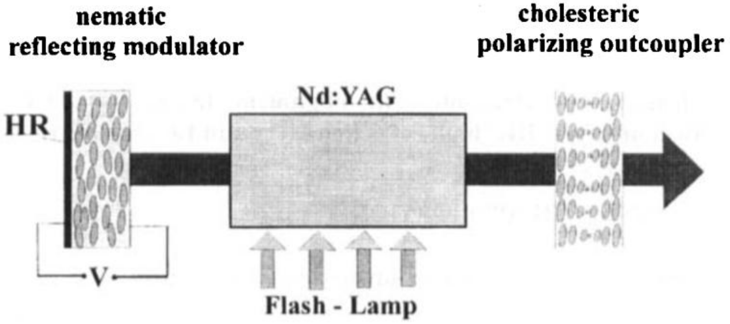
Key ingredients:

- Gain medium pumped over transparency
- An optical cavity with tunable losses
- A fast active/passive tuning method

Active Q-switching

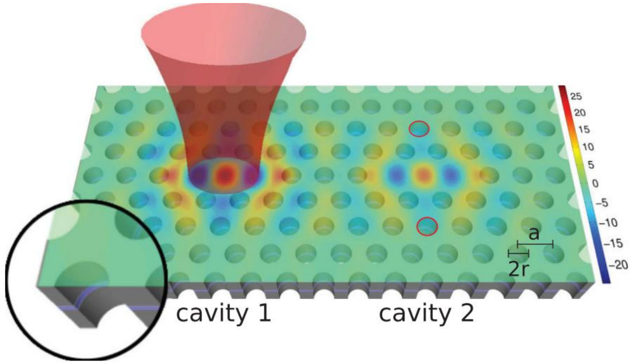


Applied Optics Vol. 1, Issue S1, pp. 103-105 (1962)

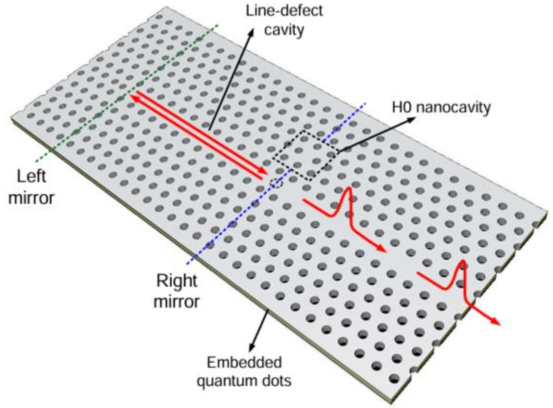


Eichler, H. J. et al., Mol. Cryst. and LC Sc. & Tech. 320(1), 89–99 (1998)

Passive Q-switching



Yacomotti et al., PRA 87, 041804(R) (2013)



Yu et al., Nat. photonics **11**, 81–84 (2017)

Q-splitting case ($\boldsymbol{\phi = 0}$) → lasing condition

Rate equations (normalized)

$$\dot{a_1} = \frac{1}{2}[-1 + (n_1 - n_{tr})(1 + i\alpha_h)]a_1 - \gamma_c a_2$$

$$\dot{a_2} = \frac{1}{2}[-1 + (n_2 - n_{tr})(1 + i\alpha_h)]a_2 - \gamma_c a_1$$

$$\dot{n_1} = R_{\mathrm{inj},1} - \frac{\mathrm{n}_1^2}{\tau_{rad}} - \frac{\mathrm{n}_1}{\tau_{nr}} - \mathrm{g}_0(\mathrm{n}_1 - \mathrm{n}_{\mathrm{tr}})|\mathrm{a}_1|^2$$

$$\dot{n_2} = R_{\mathrm{inj},2}(t) - \frac{\mathrm{n}_2^2}{\tau_{rad}} - \frac{\mathrm{n}_2}{\tau_{nr}} - \mathrm{g}_0(\mathrm{n}_2 - \mathrm{n}_{\mathrm{tr}})|\mathrm{a}_2|^2$$

Gain detuning:

$$\delta g = \frac{1}{2}(n_1 - n_2)$$

Frequency detuning:

$$\delta\omega = \frac{\alpha_h}{2}(n_1 - n_2)$$

Quasi normal modes gain:

$$g_\pm = \frac{1}{2}(-1 + \bar{n} - n_{tr}) \pm \frac{1}{2} Im\left(\sqrt{(\alpha_h + i)^2(n_1 - n_2)^2 - 4\gamma_c^2}\right)$$

$$n_1 = n_2 \quad \Rightarrow \quad g_\pm = \frac{1}{2}(-1 + \bar{n} - n_{tr}) \pm \gamma_c \Rightarrow \quad \boldsymbol{n_{th} = 1 - 2\gamma_c + n_{tr}}$$

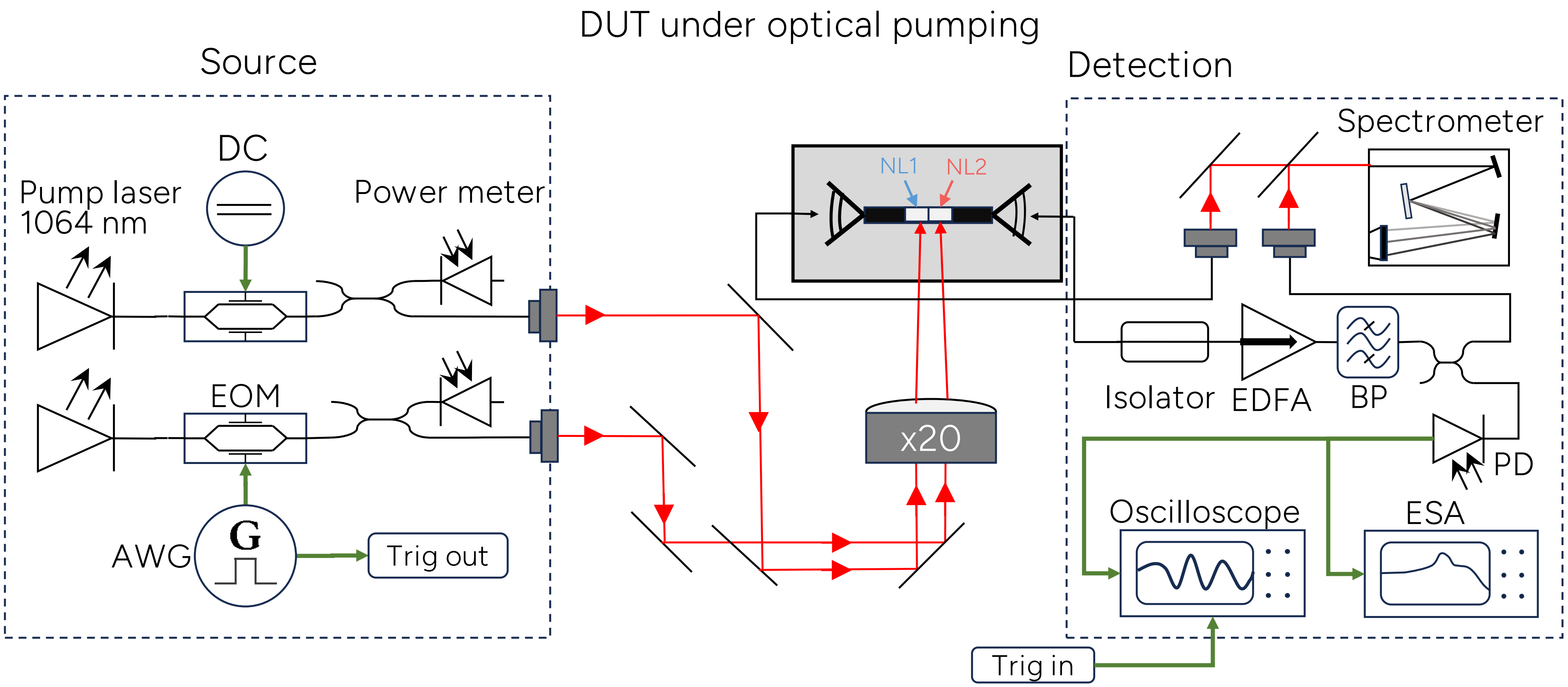
Photon lifetime
$\tau_{ph} \sim 1.1$ ps
Carrier lifetime
$\tau_{nr} \sim 3.2$ ns
Coupling rate
$\Gamma_c = 400$ GHz (→$\gamma_c = 0.45$)

1) Possible lasing emission for $n_{1,2}$ above transparency but below threshold, provided that $\delta\omega \approx 0$

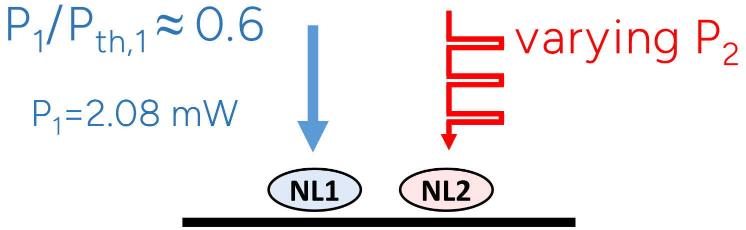
2) The other QNM gets all the losses

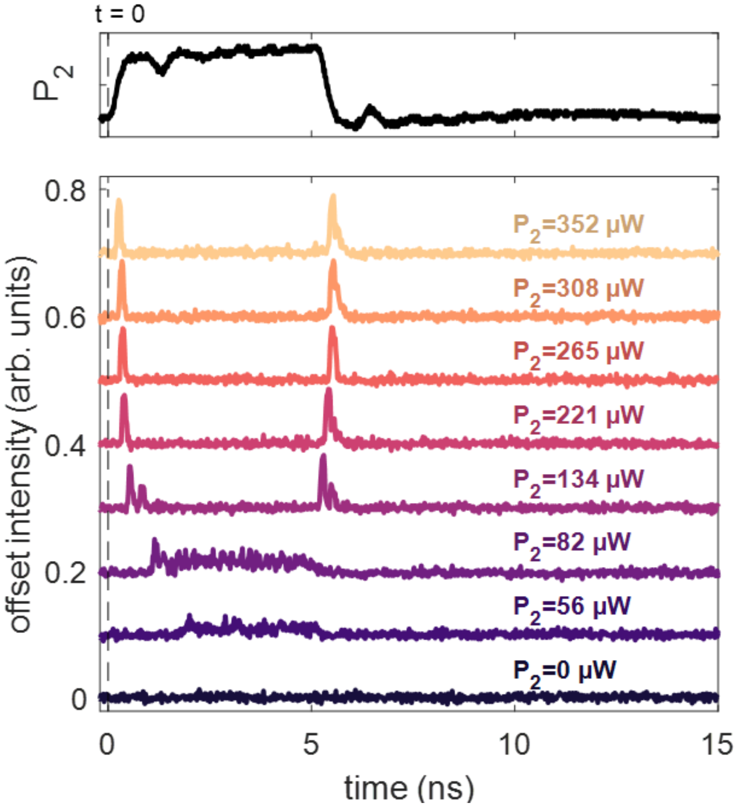
3) $\delta\omega/\delta g = \boldsymbol{\alpha_h \approx 5}$
Detuning condition can be achieved with the injection rate

Experimental setup



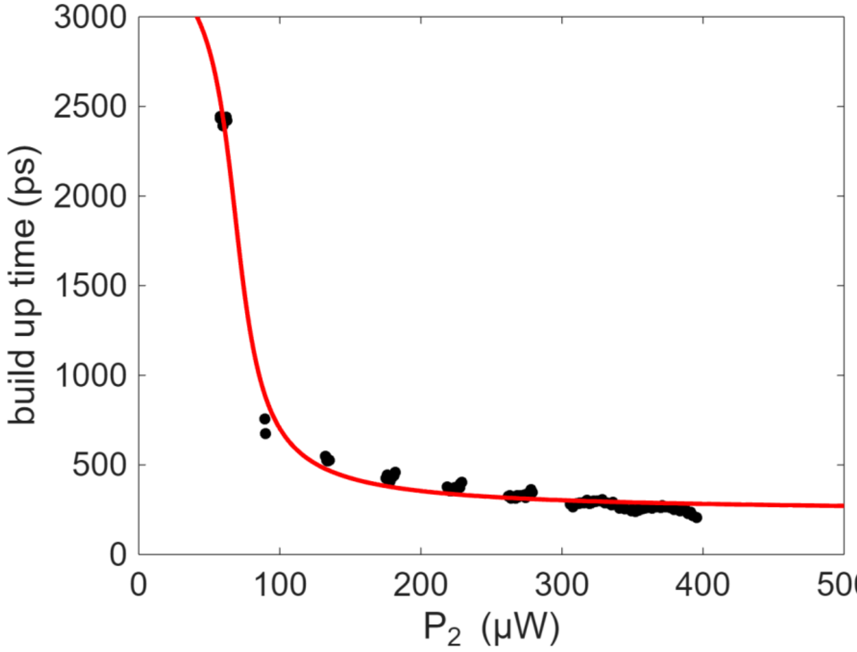
Q-switching pulses





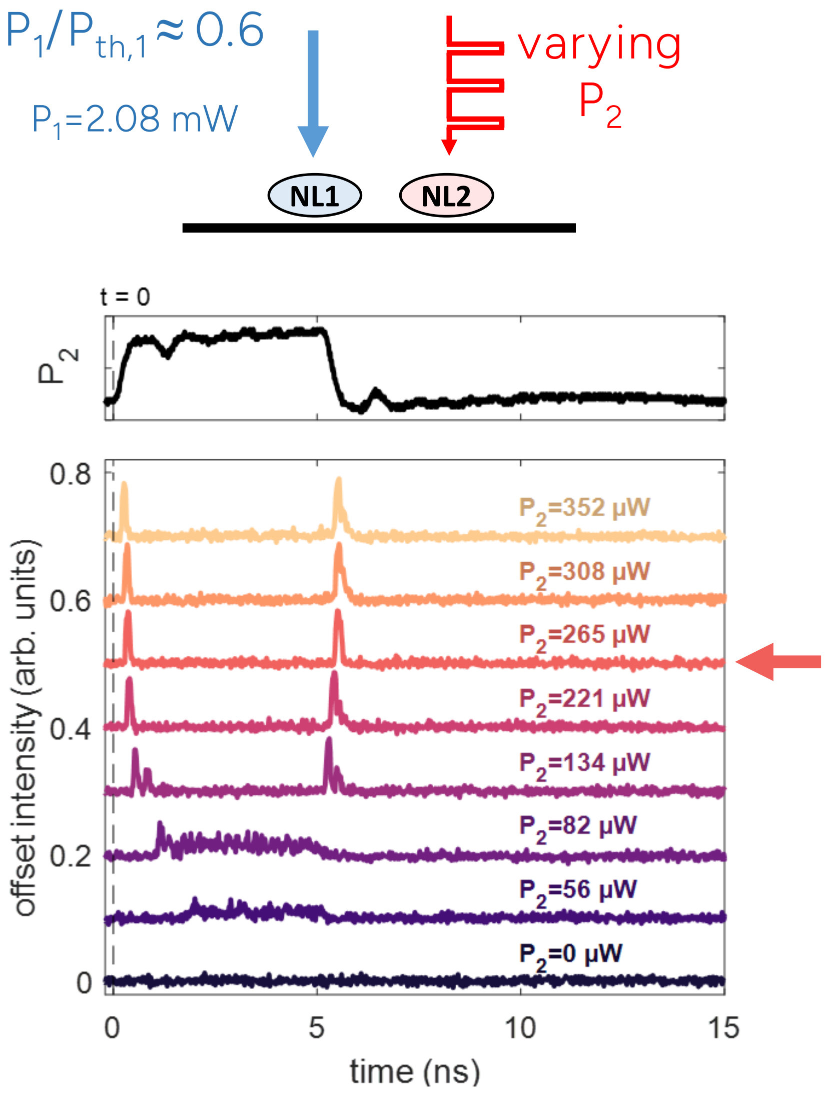
- Nanolaser 1 is pumped below threshold
- Nanolaser 2 is pumped with 5ns long, 25ps rise/fall time pulses

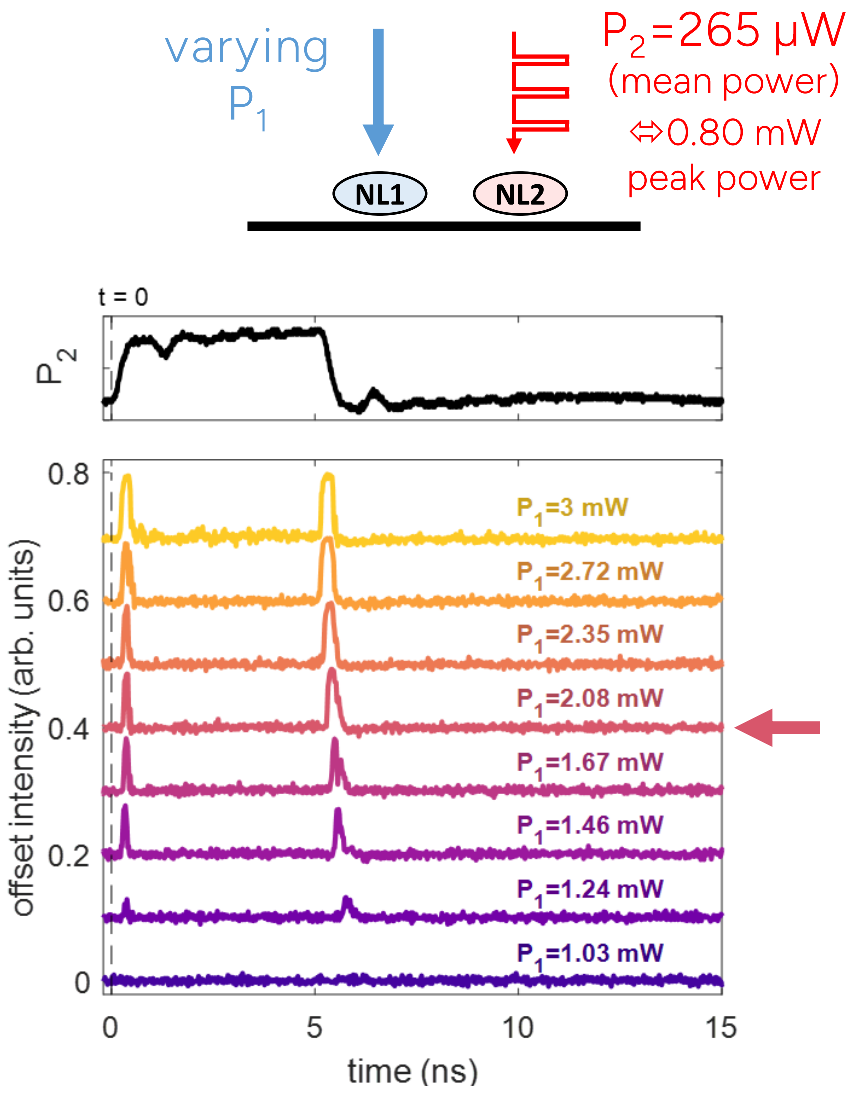
➢ Pulses are observed both at upward and downward pump changes



➢ Build-up time is fitted accordingly with the carrier density rate equation → $t_{onset} = t_0 + \int_{N_1}^{N_2} \frac{dn}{P_2 - n - \frac{n^2}{\tau_{rad}}}$

Q-switching pulses





✓ Only pumping nanolaser 2 does not work

Numerical simulations

Rate equations

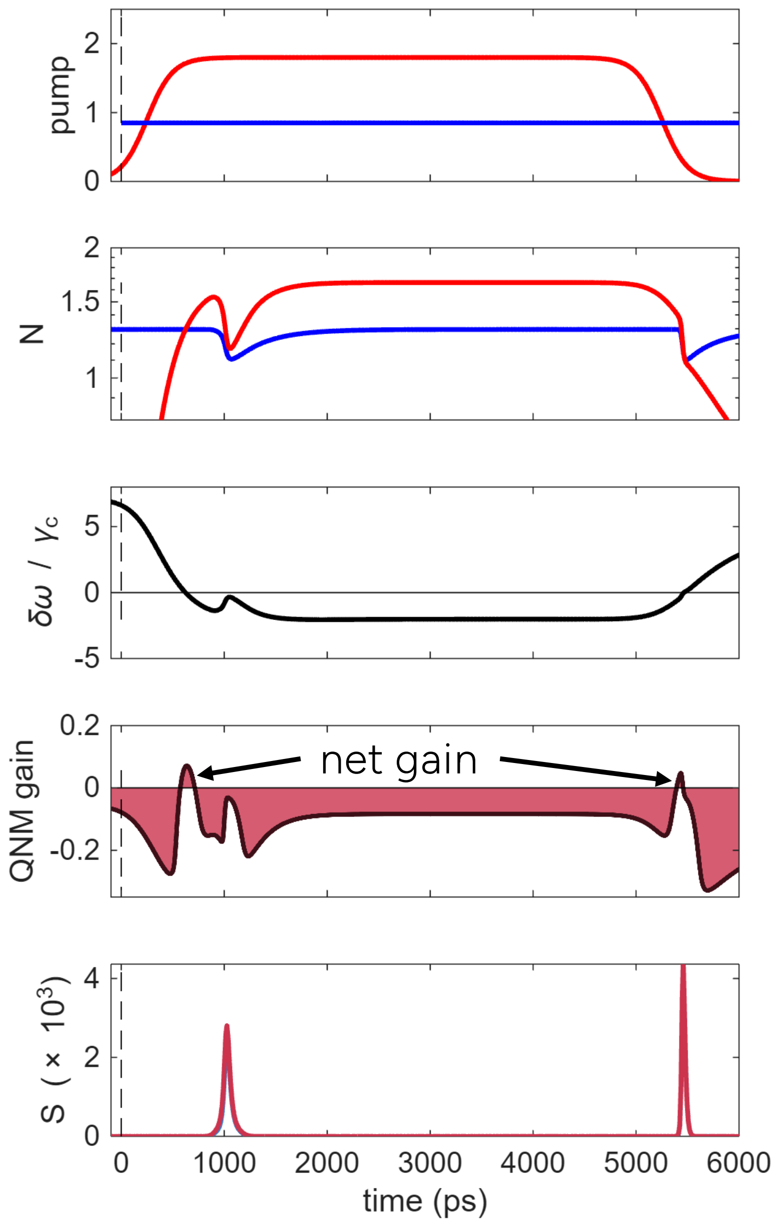
$$\dot{a_1} = \frac{1}{2}[-1 + (n_1 - n_{tr})(1 + i\alpha_h)]a_1 - \gamma_c a_2$$

$$\dot{a_2} = \frac{1}{2}[-1 + (n_2 - n_{tr})(1 + i\alpha_h)]a_2 - \gamma_c a_1$$

$$\dot{n_1} = R_{\text{inj},1}\left(1 - \frac{n_1}{n_{sat}}\right) - \frac{n_1^2}{\tau_{rad}} - \frac{n_1}{\tau_{nr}} - g_0(n_1 - n_{tr})|a_1|^2$$

$$\dot{n_2} = R_{\text{inj},2}(t)\left(1 - \frac{n_2}{n_{sat}}\right) - \frac{n_2^2}{\tau_{rad}} - \frac{n_2}{\tau_{nr}} - g_0(n_2 - n_{tr})|a_2|^2$$

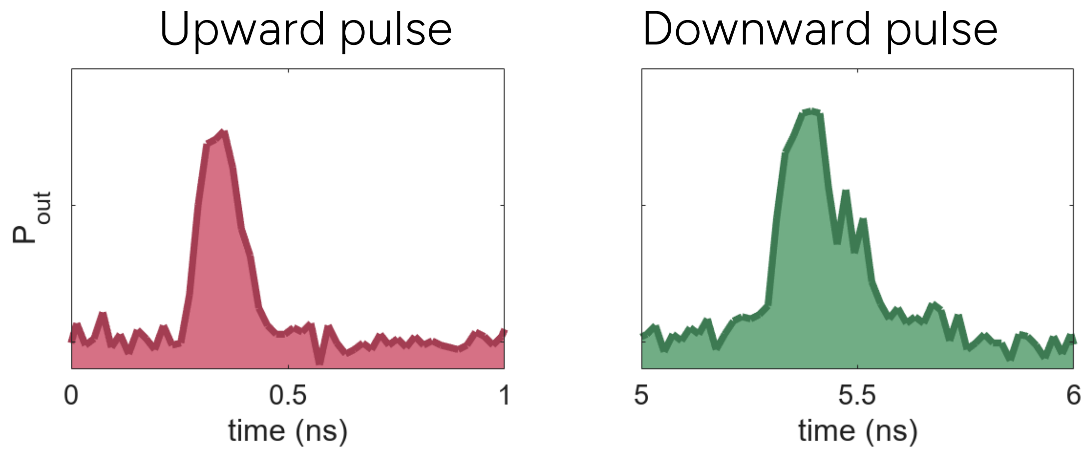
- $\delta\omega = 0$ when carrier densities cross
- Optical loss drop by Γ_c (→ Q-switching)
- Carriers are depleted leading to an optical pulse
- Same process occurs on the reverse path, but with different initial/target carrier populations.
- The other QNM (not shown here) does not contribute

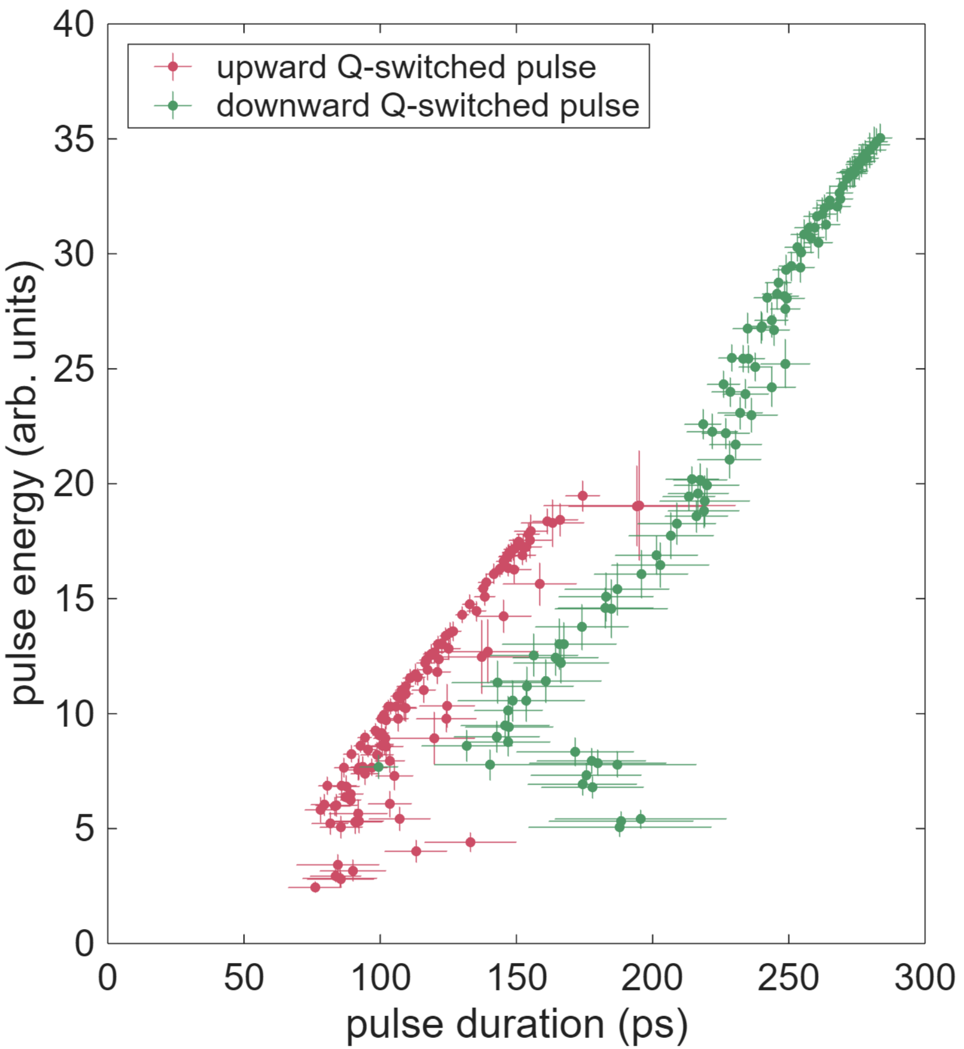


Pulse performances

- Pulse energy
  →not calibrated (yet), simulations gives $E_{pulse} \approx$ 10-100 aJ.
- Pulse width ∈[50, 300] ps

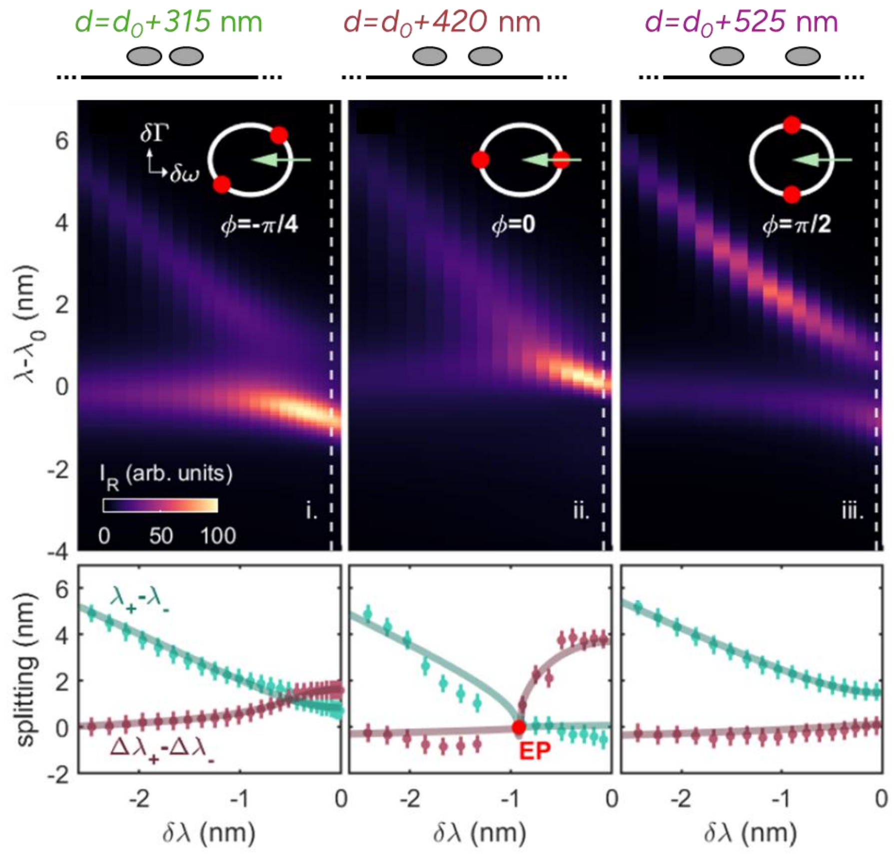
- Repetition rate
- Given by the excitation pulse width
- Limited by the carrier lifetime : (f_{max}=500 MHz, can be engineered to be >1 GHz).





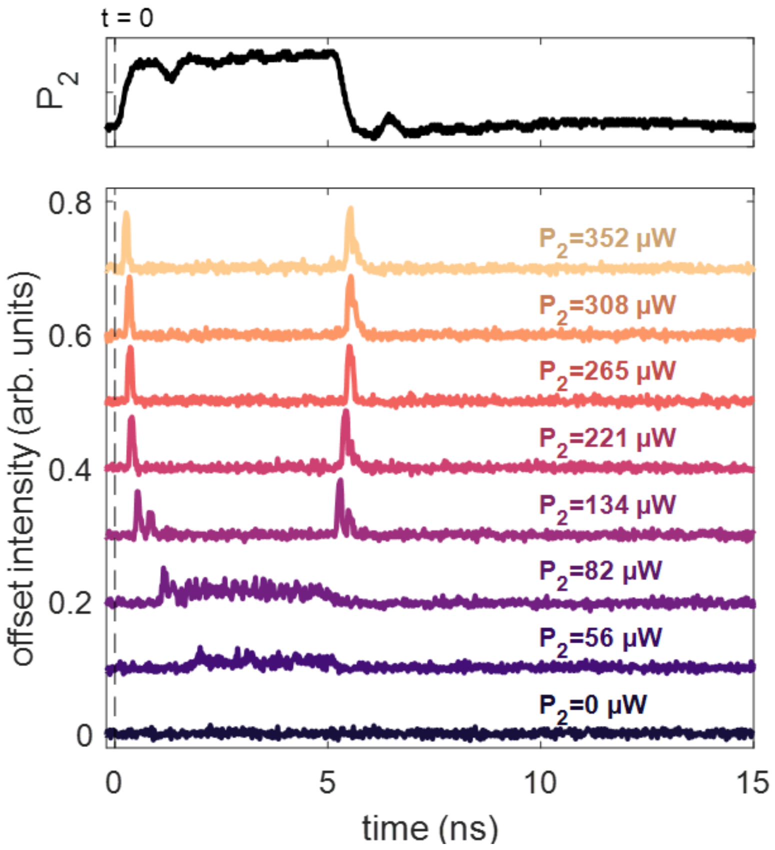
Summary

- $\boldsymbol{\phi}$-coupling between two photonic crystal nanolasers
- Q-splitting can be obtained at $\phi = 0\ [\pi]$.



Madiot *et al*, Science Advances 10,45 (2024)

- ~100 ps pulses induced by active Q-switching
- Occurs under direct-modulation of the pump power.



Acknowledgements

People

Fabrice Raineri

Guillaume Huyet

Patricia Loren

Kristian Seegert

- *Grégoire Beaudoin*
- *Konstantinos Pantzas*
- *Isabelle Sagnes*

Fundings

guilhem.madiot@univ-cotedazur.fr