

Karmenen: Lectures

①

→ Classical counterparts of Keldysh: MSR for stoch. systems, for example.

→ Keldysh formalism useful for: 1) out-of-eg. QFT

- 3) Eg. problems where Dyson's analytical cont. proves difficult. 2) Alternative to replica & supersymmetry methods in disordered & glassy systems

A) Closed Time contour:



(adiabatically!)

Heisenberg rep.

$$C(t, t') \equiv \langle 0 | \bar{T}(A(t)B(t')) | 0 \rangle$$

GS or thermal state of $H = H_0 + H_{int}$

and $|0\rangle = \hat{S}(0, -\infty) | \rangle_0$

Smearing operator.

GS of H_0

$$S(t, t') = e^{i\hat{H}_0 t} e^{-i\hat{H}(t-t')} e^{-i\hat{H}_0 t'}$$

(interaction picture!)

interaction representation

$$\hat{A}(t) = \hat{S}^\dagger(t, 0) \hat{A}(t) \hat{S}(t, 0)$$

(2)

$$A_I(t) = A(t) = e^{iH_0 t} \underbrace{A_S(t)}_{= A \text{ operator unchanged}} e^{-iH_0 t}$$

Quick reminder:

$$\langle \hat{O}(t) \rangle = \langle \psi(t) | \hat{O} | \psi(t) \rangle_S$$

$$= \langle \psi(0) | \hat{U}^\dagger(t,0) \hat{O} \hat{U}(t,0) | \psi(0) \rangle$$

$$= \langle \psi(0) | \hat{O}_H | \psi(0) \rangle$$

$$= \langle \psi(0) | \hat{U}^\dagger e^{-iH_0 t} e^{iH_0 t} \hat{O} e^{-iH_0 t} \times e^{iH_0 t} | \psi(0) \rangle$$

$$= \langle \psi(t) | \underbrace{e^{iH_0 t} \hat{O} e^{-iH_0 t}}_{\hat{O}_I(t)} | \psi(t) \rangle_I$$

$$S^\dagger(0, -\infty) = S(-\infty, 0)$$

$$\Rightarrow C(t, t') = \langle | \hat{T} \left(\hat{S}^\dagger(-\infty, 0) A(t) B(t') S(0, -\infty) \right) \rangle$$

$$= \langle | \hat{T} \left(\hat{S}(-\infty, 0) \hat{S}(0, t) A(t) \hat{S}(t, 0) S(0, t') B(t') \hat{S}(t', 0) \times S(0, -\infty) \right) | \rangle_0$$

$$\Rightarrow C(t, t') = \langle 1 | \hat{T} \left(\hat{S}(-\infty, 0) A(t) \hat{S}(t, t') B(t') \right) | 1 \rangle \quad (3)$$

$$= \langle 1 | \hat{T} \left(\hat{S}(-\infty, 0) A(t) \hat{S}(0, t) \hat{S}(t, t') \hat{S}(t', 0) B(t') \right) | 1 \rangle$$

$$= \langle 1 | \hat{T} \left(A(t) B(t') \hat{S}(-\infty, +\infty) \right) | 1 \rangle \times \langle 0, -\infty | 0 \rangle$$

now, we use that $|0, +\infty\rangle \sim |0, -\infty\rangle$, i.e. the vacua are equivalent, s.t.

$$\langle 0, -\infty | \hat{S}(-\infty, 0) = \langle 0, -\infty | \hat{S}(\infty, -\infty) \hat{S}(-\infty, 0) e^{iL}$$

$$= e^{iL} \langle 0, +\infty |$$

where

such that:

$$C(t, t') = \langle 0, -\infty | \hat{T} (A(t) B(t')) \hat{S}(0, -\infty) | 0 \rangle$$

$$= e^{iL} \langle 0, -\infty | \hat{S}(\infty, -\infty) \hat{S}(-\infty, 0) \hat{T} (A(t) B(t')) \hat{S}(0, -\infty) | 0 \rangle$$

$$= e^{iL} \langle 0, -\infty | \hat{T} (A(t) B(t')) \hat{S}(\infty, -\infty) \hat{S}(-\infty, -\infty) | 0 \rangle$$

$$= e^{iL} \langle 0, -\infty | \hat{T} (A(t) B(t')) \hat{S}(\infty, -\infty) | 0 \rangle$$

(4)

$$e^{iL} = \langle R S(-\infty, 0) \rangle$$

$${}_0 \langle 1 | \hat{S}(-\infty, 0) \times S(0, -\infty) \rangle = {}_0 \langle 1 | = 1$$

$$\Rightarrow {}_0 \langle 1 | \hat{S}(-\infty, 0) S(0, -\infty) \rangle = {}_0 \langle 1 | = e^{-iL} {}_0 \langle 1 | \hat{S}(\infty, -\infty)$$

$$\Rightarrow e^{iL} {}_0 \langle 1 | \rangle = {}_0 \langle 1 | \hat{S}(\infty, -\infty) \rangle \quad \text{vacuum bubble!}$$

Assumption (crucial):

@ $t = -\infty$ we have
no interactions (CS),

and @ $t = +\infty$ its are again switched off
adiabatically, $\Rightarrow$ since we assume a UNIQUE
CS, we know that $|0, \infty\rangle \sim |0, -\infty\rangle$.

NOT TRUE OUT OF EQUILIBRIUM.

④ $\hat{U} = \langle \hat{S}(t=0,0) \rangle$

$\langle \hat{S}(t=0,0) \times \hat{S}(0,-\infty) \rangle = \langle 1 \rangle = 1$

$\Rightarrow \langle \hat{S}(t=0,0) \hat{S}(0,-\infty) \rangle = \langle 1 \rangle = e^{iL} \langle \hat{S}(0,-\infty) \rangle$

$\Rightarrow e^{iL} \langle 1 \rangle = \langle \hat{S}(0,-\infty) \rangle$ vacuum bubble!

Assumption (crucial): @ $t = -\infty$ we have no interactions (CS),

and @ $t = +\infty$ its are again switched off adiabatically, $\Rightarrow$ since we assume a UNIQUE CS, we know that $|0, \infty\rangle \sim |0, -\infty\rangle$.

NOT TRUE OUT OF EQUILIBRIUM.

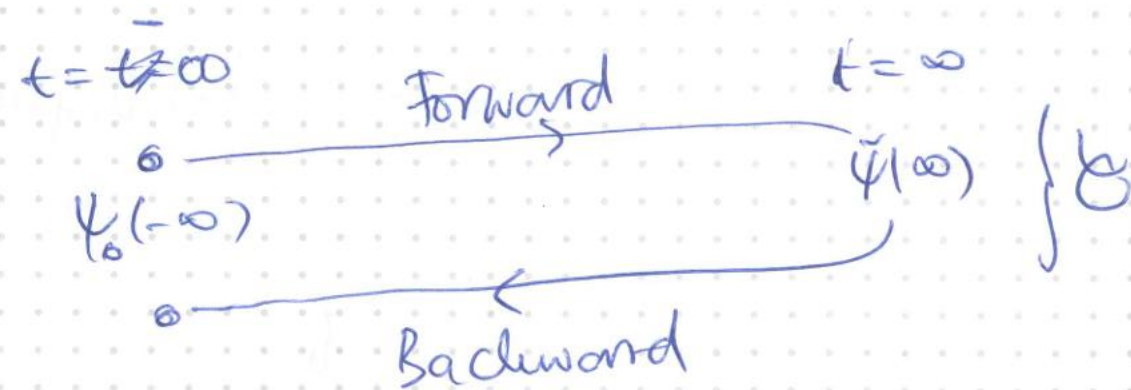
⑤ Switching (ground) procedure matters!

$\Rightarrow$ starting from $t = -\infty$ $\psi_0(-\infty)$, we find after switching, that the time-evolution is to some unpredictable state $\tilde{\psi}(+\infty) \neq \psi_0(-\infty)$.

-1 however since we trace we need to know both final & initial states!

Schwinger: take them both to be the same!

$\rightarrow$ how?



no switching on at $t = -\infty$ (past) and off in future $t = \infty$, but both happen @ same point on diff. contour branches!

$\rightarrow$ absence of information @ $t = \infty$ avoided entirely

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How does this look?

$$\begin{aligned}
 \rho(t) &= e^{iH(t-t_0)} \rho_0 e^{-iH(t-t_0)} \\
 \rho_0 &= \rho(t_0) \\
 \langle O(t) \rangle &= \text{tr} \left(\rho(t) \hat{O}(t) \right) \\
 &= \text{tr} \left(e^{iH(t-t_0)} \rho_0 e^{-iH(t-t_0)} \hat{O}(t) \right) \\
 &= \text{tr} \left(\rho_0 e^{-iH(t-t_0)} \hat{O}(t) e^{iH(t-t_0)} \right) \\
 &= \text{tr} \left(\rho_0 \hat{O}(t_0) \right)
 \end{aligned}$$

$$\langle O(t) \rangle = \text{Tr} \left(\hat{O}(t) \rho(t) \right)$$

$$\langle O(t) \rangle = \frac{\text{Tr}(\hat{O} \rho(t))}{\text{Tr}(\rho(t))} = \frac{1}{\text{Tr}(\rho(t))} \text{Tr}(\hat{O} \rho(t))$$

→ now introduce $U_{t_0 \rightarrow t} U_{t \rightarrow t_0} = \mathbb{I}$

$$\Rightarrow \frac{1}{\text{Tr}(\rho(t))} \text{Tr} \left(U_{t_0 \rightarrow t} U_{t \rightarrow t_0} \hat{O} U_{t_0 \rightarrow t} \rho(t_0) \right)$$

$$= \frac{1}{\text{Tr}(\rho(t))} \text{Tr} \left(U_{t_0 \rightarrow t} \hat{O} U_{t \rightarrow t_0} \rho(t_0) \right)$$



✓

⑥ how does this look?

$$\begin{aligned}
 \rho(t) &= e^{iH(t-t_0)} \rho_0 e^{-iH(t-t_0)} \\
 \rho_0 &= \rho(t_0) \\
 \langle O(t) \rangle &= \text{tr}(\rho(t) \hat{O}(t)) \\
 &= \text{tr}(e^{iH(t-t_0)} \rho_0 e^{-iH(t-t_0)} \hat{O}(t)) \\
 &= \text{tr}(\rho_0 e^{-iH(t-t_0)} \hat{O}(t) e^{iH(t-t_0)}) \\
 &= \text{tr}(\rho_0 \hat{O}(t_0))
 \end{aligned}$$

So $\langle O(t) \rangle = \text{tr}(\hat{O}(t) \rho(t))$

$$\langle \bar{O}(t) \rangle = \frac{\text{Tr}(\hat{O} \bar{\rho}(t))}{\text{Tr}(\bar{\rho}(t))} = \frac{1}{\text{Tr}(\bar{\rho}(t))} \text{Tr}(\hat{U}_{\text{tot}} \hat{O} U_{\text{tot}}^\dagger \bar{\rho}(-\infty))$$

→ now introduce $U_{\text{tot}} U_{\text{tot}}^\dagger = \mathbb{I}$

$$\Rightarrow \frac{1}{\text{Tr}(\bar{\rho}(t))} \text{Tr}(U_{-\infty}^\dagger U_{t_0} U_{\text{tot}} \hat{O} U_{t_0}^\dagger U_{-\infty} \bar{\rho}(-\infty))$$

$$= \frac{1}{\text{Tr}(\bar{\rho}(t))} \text{Tr}(U_{-\infty}^\dagger U_{t_0} U_{\text{tot}} \hat{O} U_{t_0}^\dagger U_{-\infty} \bar{\rho}(-\infty))$$



$$\rightarrow Z[0] = \frac{\text{tr}(\hat{U}_0 \bar{\rho}(-\infty))}{\text{tr}(\bar{\rho}(-\infty))} = 1 \text{ always!}$$

observables can now be calculated by adding

sources $H_0 = H \pm \frac{\eta(t) \hat{O}(t)}{Z}$ where "+" plus branch

and "-" minus branch

on average per convention.

Since t_0 is non-synchronous along the branches,

$Z(\eta)$ is not zero,

$$\rightarrow \langle \bar{O}(t) \rangle = \frac{\delta \ln(Z)}{\delta \eta(t)} \Big|_{\eta=0} \text{ since } Z[0] = 1$$

(Absence of $\frac{1}{Z}$ makes this technique much simpler for systems with quenched disorder.)

⑧ Algebra of Green's functions

First: $Z = \int D\bar{\Phi} D\phi e^{iS[\bar{\Phi}, \phi]}$

"

$$e^{\int dt \int dx \bar{\Phi} \hat{H} \phi}$$

$$S[\bar{\Phi}, \phi] = \sum_{j=2}^{2N} \left[i\bar{\Phi}_j \left(\frac{\phi_j - \phi_{j-1}}{\delta t_j} - \omega_0 \bar{\Phi}_j \phi_{j-1} \right) \delta t \right]$$

$$+ i\bar{\Phi}_1 (\phi_1 - p(\omega_0) \phi_{2N}) \rightarrow \text{ef. in tunnel ef.}$$

$p(\omega_0) = e^{\beta(\omega_0 - \mu)}$

important! contains info about
distribution function!

$$\hookrightarrow G(t, t') = -i \int D\bar{\Phi} D\phi e^{iS[\bar{\Phi}, \phi]} \phi(t) \bar{\phi}(t')$$

and comes from $\langle \phi_1 | \hat{\rho}_0 | \phi_{2N} \rangle$
matrix element.

$$= \overline{\bar{\Phi}_1 \phi_{2N}} p(\omega_0)$$

$$\text{or } \hat{\rho}_0 \sim e^{-\beta(\omega_0 - \mu)}$$

tunnel density matrix!

Banyan Chapter 2: Time reversal and evolution eq.

Lagrangian function of a classical system of point like interacting particles:

$$L = \sum_{i=1}^N \left[\frac{m \dot{x}_i^2}{2} - U(x_i) \right] - \frac{1}{2} \sum_{i,j} V(|x_i - x_j|)$$

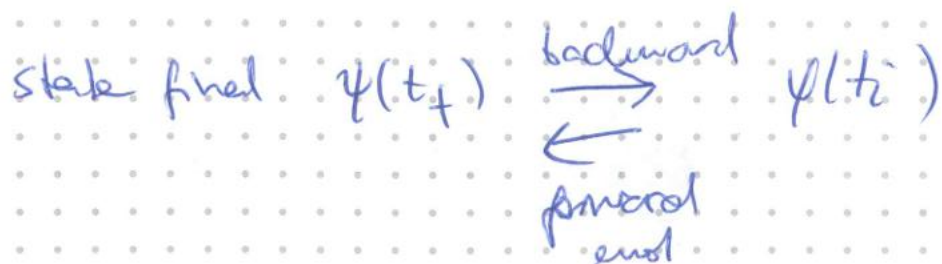
invariant under $t \rightarrow -t$, since $\dot{x} = \frac{dx}{dt} \rightarrow -\dot{x}$

and therefore $L(t \rightarrow -t)$ invariant.

$\rightarrow$ consequence of the fact that $F = -T U$

and thus $F(-t) = F(t)$ as well.

$\rightarrow$ evolution is reversible.



(Poincaré: after a finite time t^* system returns to vicinity of initial state in phase space Check!)

In Q.M.S unitary time evol: $H^+ = H$: $\psi(t)$ solution to SEs $\Rightarrow \psi^*(-t)$ also sol. (if $\partial_t \psi = H\psi$)

(7)

→ thermalization is not reversible! ⚡ issue?

Example: Ohms law!

$$\boxed{\vec{j} = \sigma \vec{E}}$$

✓ current density

$$\sim \vec{v} = -\vec{v}(-t)$$

intensity of applied field

$$\sim \vec{F} \sim \ddot{x} = \ddot{x}(-t)$$

↳ Newtonian mechanics (2nd law)

$$\ddot{x} \sim \dot{p} \propto F \sim E$$

⚡

⇒ need a non mechanical (friction) force:

$$\ddot{x}(t) = eE - \kappa \dot{x}(t)$$

⇒ asymptotic motion is $\ddot{x}(t) = 0$

$$\Rightarrow \dot{x}(t) = \dot{x}^* = \frac{eE}{\kappa}$$

↳ never possible to "solve" in closed systems.

But: systems never closed! 😊

ization is not reversible! $\Downarrow$ issue?

Ohms law!

$$\boxed{\vec{j} = \sigma \vec{E}}$$

intensity of applied field

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 $\sim \vec{v} = -\vec{v}(-t)$

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Newtonian mechanics (2nd law)

$$\ddot{x} \sim \dot{p} \propto F \sim E$$

need a non mechanical (friction) force,

$$m\ddot{x}(t) = eE - \kappa \dot{x}(t)$$

asymptotic motion is $m\ddot{x}(t) = 0$

$$\Rightarrow \ddot{x}(t) = 0 \quad \ddot{x} = \frac{eE}{\kappa}$$

never possible to "solve" in closed systems.

never closed! $\odot$

→ To study thermalization we can study "open systems", or a "bath" w/ ∞ many dof and ∞ vol. w/ finite density.

→ "thermostat" when this is in eq. plays central role in understanding equilibration.

→ as an example ("classical") we study the 1D Caldeira & Leggett model.

(caveat: superfluids & SCs... here not our case!)

Chapter 3: Caldeira & Leggett 1983: motion of e^- ($m=1, e=1$) of coord $x(t)$ interacting w/ acoustic phonons & constant electric field E .

$$\sim Q_q, \omega_q = c|q|, c \sim \text{sound velocity}$$

$q \in [0, q_D]$ Debye wave-vector

$$= \sqrt{\frac{c\kappa}{\pi}} \omega_q$$

$$\mathcal{L} = \frac{1}{2} \dot{x}^2 + Ex + \frac{1}{2} \int_{|q| < q_D} dq \left\{ \dot{Q}_q^2 - \left(\frac{F_q}{\omega_q} x - \omega_q Q_q \right)^2 \right\}$$

(4)

EL. eq. s.

$$\ddot{Q}_q = \omega_q \left(\frac{F_q}{\omega_q} x - \omega_q Q_q \right)$$

$$\ddot{x} = \varepsilon - \int_{|q| < q_D} dq \frac{F_q}{\omega_q} \ddot{Q}_q$$

$$(S = \int dx dt + \mathcal{L}(x, \dot{x}))$$

Deriv. EL. eq. s. for $(q, \dot{q})$ from $\delta S = 0$.

$$\frac{\delta \mathcal{L}}{\delta q} = \frac{\partial}{\partial t} \frac{\delta \mathcal{L}}{\delta \dot{q}}$$

$$(1) \rightarrow \frac{\delta \mathcal{L}}{\delta Q_q} = \frac{\partial}{\partial t} \frac{\delta \mathcal{L}}{\delta \dot{Q}_q}$$

$$(2) \frac{\delta \mathcal{L}}{\delta x} = \frac{\partial}{\partial t} \frac{\delta \mathcal{L}}{\delta \dot{x}}$$

$$(1) - \int_{|q| < q_D} (\omega_q^2 Q_q - F_q x) \delta Q_q$$

$$= \int_{|q| < q_D} \ddot{Q}_q \delta Q_q$$

$$(2) \varepsilon + \int_{|q| < q_D} \left(F_q Q_q - \frac{F_q^2}{\omega_q^2} x \right)$$

$$= \ddot{x}$$

$$\Rightarrow \ddot{Q}_q = -\omega_q^2 Q_q + F_q x \quad \& \quad \ddot{x} = \varepsilon + \int_{|q| < q_D} \left(F_q Q_q - \frac{F_q^2}{\omega_q^2} x \right)$$

$$= -\omega_q^2 \left(Q_q - \frac{F_q}{\omega_q^2} x \right)$$

$$\Rightarrow = \varepsilon - \int_{|q| < q_D} \frac{F_q}{\omega_q^2} \ddot{Q}_q$$

Initial cond: $\hat{x}(0) \wedge Q_q(0) = \dot{Q}_q(0) = \dot{x}(0) = 0$
 $\parallel$
 V_0

⑤

$\Rightarrow$ Solution: if $F_q = 0$

$$\ddot{Q}_q(t) = Q_{\text{free}} \times \cancel{Q_{\text{comp}}}$$

where Q_{free} is solution of $\ddot{Q}_q(t) = -\omega_q^2 Q_q$

i.e. we take $Q_q(t) = A e^{it\omega_q} + B e^{-it\omega_q}$

but since $Q_q(0) = 0$ it has to be $\sin(\omega_q t)$.

and since $\dot{Q}_q(0) = 0$ also we need..

Thus $Q_{\text{free}} = \alpha \sin(\omega_q t)$.

then we need the coupled term.

$$\begin{aligned} \text{formally: } \ddot{Q}_q &= \cancel{[Q_{\text{free}} Q_{\text{comp}}]} \\ &= \cancel{\ddot{Q}_{\text{free}} Q_{\text{comp}}} + \cancel{Q_{\text{free}} \ddot{Q}_{\text{comp}}} \\ &\quad + \cancel{2\dot{Q}_{\text{free}} \dot{Q}_{\text{comp}}} + \cancel{Q_{\text{free}}} \\ &= \cancel{-\omega_q^2 Q_{\text{free}} Q_{\text{comp}}} + \cancel{Q_{\text{free}} \ddot{Q}_{\text{comp}}} \end{aligned}$$

since $F_q \neq 0$ what do we do?

so solution: $Q = \int_0^t dt' Q_{\text{free}}^{(t-t')} F(t')$

$$\rightarrow \ddot{Q} = -\omega_q^2 Q + \cancel{Q} \cancel{Q} \left(\underbrace{\int_0^{t+\Delta t} dt' Q_{\text{free}} - \int_0^t dt' Q_{\text{free}}}_{\Delta t} \right)$$

$$\Rightarrow \ddot{Q} = -\omega_q^2 Q + \int_0^t dt' \left[Q(t-t') f(t') \right]_{\text{free}}$$

$$= -\omega_q^2 Q + \left[Q(t-t') f(t') \right]_{\text{free}} \Big|_{t'=0}$$

$$\Rightarrow \cancel{f(t')} = \cancel{F_q} \quad f(t') = \beta \kappa \cdot \vec{E}_q$$

$$\cancel{Q}_{\text{free}} = \alpha \sin(\omega_q(t-t'))$$

$$\Rightarrow \alpha \times \beta \vec{E}_q = F_q \quad \text{Con later}$$

$$\Rightarrow Q = \alpha \beta \int_0^t dt' \sin(\omega_q(t-t')) \kappa(t')$$

$$\text{leads to: } \ddot{Q} = F_q \left(\int_0^t dt' \kappa(t') \cos(\omega_q(t-t')) + \kappa(0) \cos(\omega_q t) \right)$$

$$= F_q \int_0^t dt' \kappa(t') \cos(\omega_q(t-t'))$$

$$\Rightarrow \ddot{x} = \mathcal{E} - \int_{|q| < q_D} \frac{F_q}{\omega_q^2} Q_q = \mathcal{E} - \int_{|q| < q_D} \frac{F_q^2}{\omega_q^2} \int_0^t dt' \kappa(t')$$

$$F_q^2 = \frac{CK}{\pi} \omega_q^2$$

$$\Rightarrow \ddot{x} = \mathcal{E} - \int_{|q| < q_D} \frac{CK}{\pi} \int_0^t dt' \kappa(t') \cos(\omega_q(t-t'))$$

$$= \frac{-2}{c} \int_0^{\omega_D} d\omega_q$$

⑦

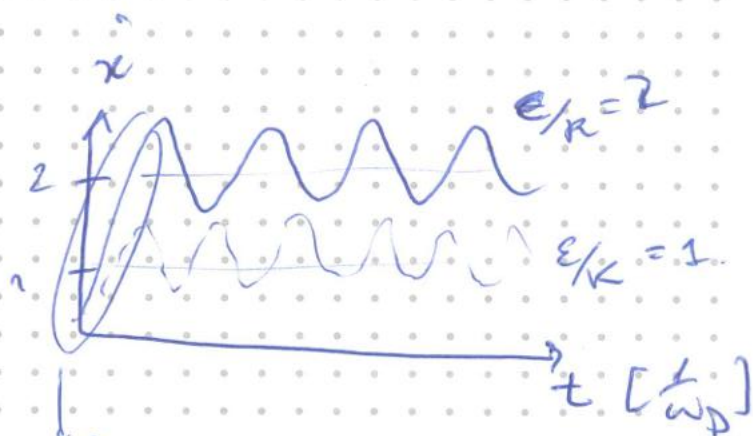
$$\Rightarrow \ddot{x} = \mathcal{E} - \frac{2}{\pi} K \int_0^t dt' \underbrace{\frac{\sin(\omega_D(t-t'))}{t-t'}}_{\text{non-local in time memory, price to pay to eliminate the phonon degrees of freedom}} \ddot{x}(t')$$

non-local in time memory, price to pay to eliminate the phonon degrees of freedom.

→ phenomenology:

oscillates

around $\frac{\mathcal{E}}{K}$



linear as expected by accel. in $\mathcal{E}$ -field.

since $\int_0^\infty du \frac{\sin(u)}{u} = \frac{\pi}{2}$

asymptotic limit of

$$\ddot{x} = \mathcal{E} - \frac{2}{\pi} K \int_0^t dt' \underbrace{\frac{\sin(\omega_D(t-t'))}{t-t'}}_{\text{non-local in time memory, price to pay to eliminate the phonon degrees of freedom}} \ddot{x}(t')$$

satisfies

$$0 = \mathcal{E} - \frac{2}{\pi} K \left(\frac{1}{2} \dot{x} \right)^{\pi/2}$$

$$\Rightarrow \dot{x} = \frac{\mathcal{E}}{K} \quad \text{B}$$

(8)

apart from oscillations: Ohmic behaviour,

and since $\frac{\sin(\omega_0(t+t'))}{t-t'}$ is peaked @ $t=t'$

we can also find that to leading order

$$\ddot{x}(t) = -\varepsilon - \frac{2}{\pi} \kappa \dot{x} \quad , \text{ i.e. - the}$$

Ohmic like damping.

Chapter 5: SEq. vs. Heisenberg Eq.

in qphys, not so straightforward to

get from Heisenberg $\rightarrow$ dissipative.

(many non controlled approxs.)

QM: finite, (closed) isolated systems

$$\text{SEq: } i\hbar \partial_t \psi = H\psi, \quad \int_{\mathcal{K}} |\psi|^2 = 1.$$

(reversible evolutions)

irreversible processes: e.g. not on very "small" basis,
but which mix microscopic elements & irreversibility.

enter: Master equations (very successful)
(or rate eqs)

" Master eq. for q state k expresses the loss & gain balance due to transitions between the unperturbed states of the system caused by the interaction (with a reservoir usually)

$$\dot{P}_k(t) = - \sum_k \{ W_{k \leftarrow k'} P_{k'}(t) - W_{k' \leftarrow k} P_k(t) \}$$

$$\sum_k P_k(t) = 1$$

transition rates: "golden rule" of QM

int. between system & res.

$$W_{k \leftarrow k'} = \frac{2\pi}{\hbar} \sum_{\alpha, \alpha'} \rho_{\alpha, \alpha'} |\langle \alpha, k | H_{int} | \alpha', k' \rangle|^2 \times \delta(E_k + E_{\alpha} - E_{k'} - E_{\alpha'})$$

prob. of a given reservoir state

→ leads to thermalization!

But also a paradox: $P_k(0) = \delta_{k, k_0}$

→ $\sum_k W_{k \leftarrow k_0} = \dot{P}_{k_0}(t)|_{t=0}$ for $k \neq k_0$

(10) however, derivation (textbook) of golden rule:
 $c_{k,\alpha}(t)$ prob. amplitude of system/reservoir state

$$c_{k,\alpha}(0) = \delta_{k,k_0} \delta_{\alpha,\alpha_0}$$

(then)

$$W_{k,\alpha,k_0,\alpha_0} = \lim_{t \rightarrow \infty} \frac{d}{dt} |c_{k,\alpha}(t)|^2$$

since

→ $W_{k,k_0} = \sum_{\alpha,\alpha_0} P_{\alpha_0} W_{k,\alpha,k_0,\alpha_0}$ & $P_k(t) = \sum_{\alpha} P_{\alpha} |c_{k,\alpha}(t)|^2$

So then dilemma: does W_{k,k_0} coincide with

$$\left. \frac{d}{dt} P_k(t) \right|_{t=0} \quad \text{or} \quad @ \quad \left|_{t=\infty} ?$$

only possible if t is both short & long enough!

So naive derivation (textbook, look up!)

2-level fermion system; coupled to a phonon bath w/ continuous spectrum.

⑪

$$H = \epsilon_1 a_1^\dagger a_1 + \epsilon_2 a_2^\dagger a_2 + \sum_{\alpha} (\hbar \omega_{\alpha} b_{\alpha}^\dagger b_{\alpha} + g_{\alpha} (a_1^\dagger a_2 + a_1 a_2^\dagger) b_{\alpha}^\dagger)$$

$$\text{and } f_n = \langle a_n^\dagger a_n \rangle \\ = \text{tr}(\rho a_n^\dagger a_n), \quad n=1,2$$

$$\begin{aligned} (\rightarrow \text{Heisenberg: } i\hbar \partial_t f_1 &= -\langle [H, a_1^\dagger a_1] \rangle \\ &= \sum_{\alpha} g_{\alpha} (\underbrace{\langle a_2^\dagger a_1 b_{\alpha} \rangle_{\text{coll}} - \langle a_1^\dagger a_2 b_{\alpha}^\dagger \rangle_{\text{coll}}}) \end{aligned}$$

collisions.

$$\text{Co it } \frac{\partial}{\partial t} \langle a_2^\dagger a_1 b_{\alpha} \rangle \Big|_{\text{coll}} = -\langle [H, a_1^\dagger a_1 b_{\alpha}] \rangle$$

$$= (\epsilon_2 - \epsilon_1 - \hbar \omega_{\alpha}) \langle a_2^\dagger a_1 b_{\alpha} \rangle$$

$$+ \sum_{\alpha'} g_{\alpha'} \langle (a_2^\dagger a_1 b_{\alpha'} + a_1^\dagger a_2 b_{\alpha'}^\dagger) b_{\alpha}^\dagger \rangle$$

new more complicated
collisions appear
@ each order!

$$\rightarrow \text{decouple: } \langle xyzw \rangle = \langle xy \rangle \langle zw \rangle$$

$$\text{or replace } \langle b_{\alpha}^\dagger b_{\alpha} \rangle = N_{\alpha}^{\text{BE}} + \langle x^\dagger x \rangle \langle y^\dagger y \rangle + \dots$$

behaviors: Ohmic behaviour,

$$\frac{\sin(\omega_0(t-t'))}{t-t'} \text{ is peaked @ } t=t'$$

find that to leading order

$$= \varepsilon - \frac{2}{\pi} \kappa x \dot{x} \text{, i.e. the}$$

damping.

SEq. vs. Klatet Eq.

not so straightforward to
believe $\rightarrow$ dissipative.
(subleading approx.)

e. (closed) isolated systems

$$i\hbar \partial_t \psi = H \psi, \quad \int_{\mathcal{K}} |\psi|^2 = 1$$

(functions)

(10) however, derivation (textbook) of golden rule

$C_{k,\alpha}(t)$ prob. amplitude of system/reservoir state

$$C_{k,\alpha}(0) = \delta_{k,k_0} \delta_{\alpha,\alpha_0}$$

$$\text{(then)} \quad W_{k,\alpha,k_0,\alpha_0} = \lim_{t \rightarrow \infty} \frac{d}{dt} |C_{k,\alpha}(t)|^2$$

since

$$\rightarrow W_{k,k_0} = \sum_{\alpha,\alpha_0} P_{\alpha_0} W_{k,\alpha,k_0,\alpha_0} \quad \& \quad P_{\alpha}(t) = \sum_{k,\alpha_0} P_{\alpha_0} |C_{k,\alpha}(t)|^2$$

So then dilemma: does W_{k,k_0} coincide with

$$\left. \frac{d}{dt} P_{\alpha}(t) \right|_{t=0} \text{ or } @ \left. \right|_{t=\infty} ?$$

only possible if t is both short & long
enough!

So naive derivation (textbook, look up!)

2-level fermion system, coupled to a phonon bath
w/ continuous spectrum.

(12)

Co. eg.: $\langle a_1^\dagger a_1 a_2^\dagger a_2 b_\alpha b_\alpha^\dagger \rangle = f_1 (1 - f_2) (N_\alpha + 1)$

ignored many correlations induced by int's)

$\rightarrow \langle a_2^\dagger a_1 b_\alpha \rangle - \langle a_1^\dagger a_2 b_\alpha \rangle$

$= -\frac{2ig_\alpha}{\hbar} \text{Re} \int_{t_0}^t dt' e^{\left\{ \frac{(\epsilon_2 - \epsilon_1 - \hbar\omega_\alpha)}{\hbar} (t - t') \right\}}$
 $\times (f_1(t') (1 - f_2(t')) (N_\alpha + 1) - f_2(t') (1 - f_1(t')) N_\alpha)$

↑
vanishing initial conditions

Co $\partial_t f_1(t) = -\frac{2}{\hbar^2} \sum_\alpha g_\alpha^2 \int_{t_0}^t dt' \cos\left(\frac{(\epsilon_2 - \epsilon_1 - \hbar\omega_\alpha)}{\hbar} (t - t')\right)$
 $\times \left(f_1(t') (1 - f_2(t')) (N_\alpha + 1) - f_2(t') (1 - f_1(t')) N_\alpha \right)$

$\rightarrow$ introduce $\chi(\omega) = \frac{1}{\hbar} \sum_\alpha g_\alpha^2 \delta(\omega_\alpha - \omega)$

$\rightarrow \partial_t f_1(t) = -\frac{2}{\hbar^2} \int d\omega \chi(\omega) I(t_0, t, \omega)$

(13) Making assumptions on the 'depth' of the memory, we can say that

$f_1(t)$ & $f_2(t)$ vary slowly

during times inversely prop to the spectral width of $M(\omega)$, so we can replace

$$f_{1/2}(t') = f_{1/2}(t).$$

then

$$\partial_t f_1(t) = \frac{2\Omega}{\hbar} M\left(\frac{\epsilon_1 - \epsilon_2}{\hbar}\right)$$

$$\times \left\{ f_1(t)(1 - f_2(t))(N+1) - f_2(t)(1 - f_1(t))N \right\}$$

i.e. a rate function...

important approx's (Marksonian)

(1) Decoupling of higher order corrs.

(2) eq. of phonon modes

(3) approx. of null memory depth.

(5 can be relaxed by having a finite lifetime memory Kernel.

like classical, but with approx's QM approx's like (1)