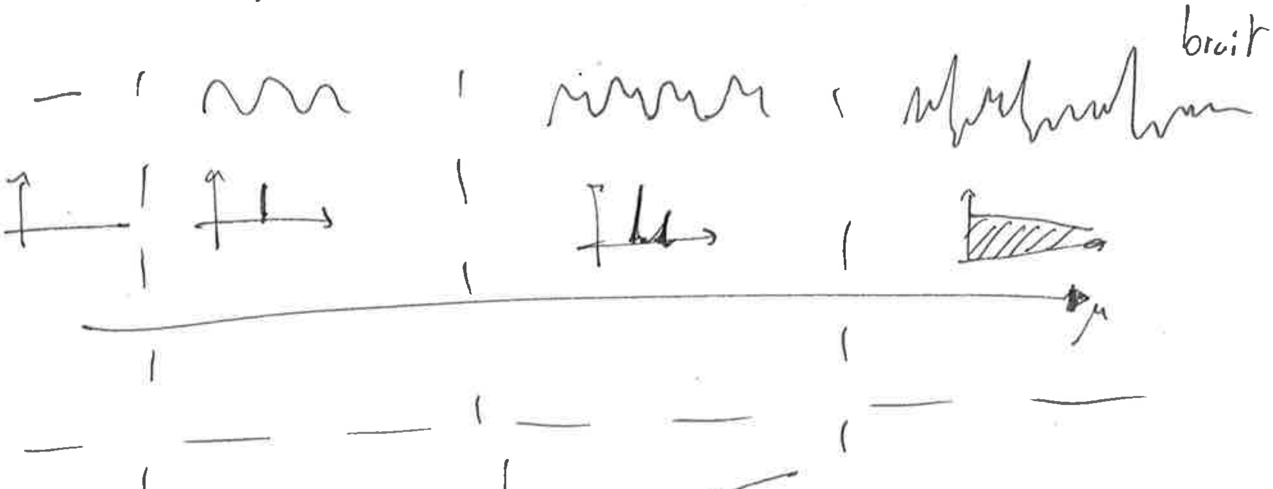
I

Système : paramètre de contrôle

↳ augmente : "énergie" : comportement + complexe
+ désordonné

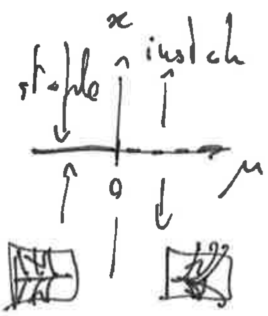
$\dot{\vec{x}} = f(\vec{x})$ $\vec{x}$ vecteur sys. dyn.



bifurcations : changement qualitatif de comportement

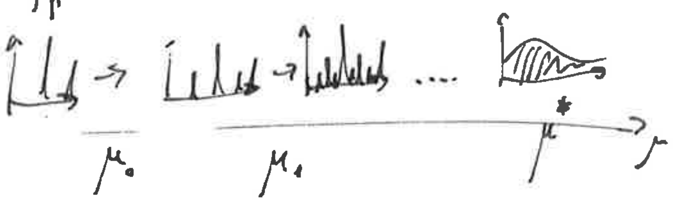
* brutal ?
  discontinu ?

ex : $\dot{x} = \mu x$



---

exemple : doublement période : un oscar devient fou



Applications

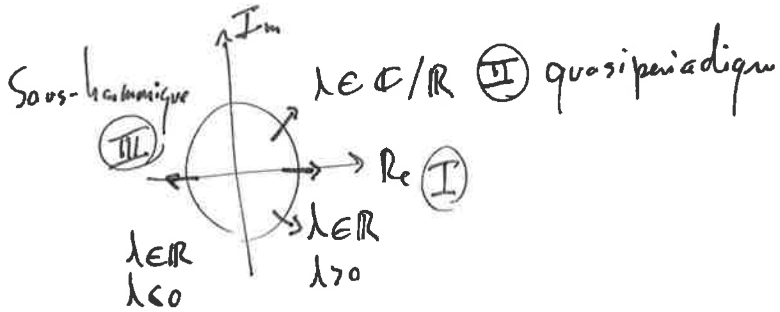
- méca flu (R–B)
- bio φ (cardiaque)
- χ (réactions oscillantes)
- €co
- optique (laser)
- électronique (diode NL)

...

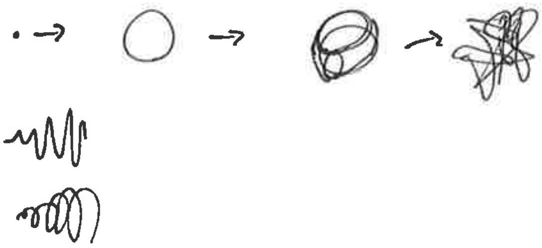
Déstabilisat°

$\dot{x} = \lambda x$

$\uparrow \lambda \Leftrightarrow e^{\lambda T}$



Fun fact : quasiharmonique (hopf



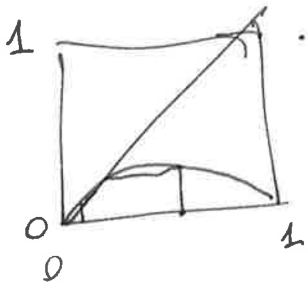
Combien pour chaos?

BREF

Rossler PDC dans ce s'intéresse à ça.

Mapping (Réduct° de dimensionalité : 3D continu
↳ 1D discret.

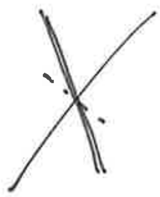
III



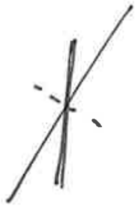
$\cdot\ f(x) = r\,x\,(1-x)$

stable

$|f'| < 1$



$f' < -1$



$f' > 1$

python

Itérat° d'endomorphismes et groupe de renormalisat°.

Pierre

Charles Coullet & Tresser → 1978

Feigenbaum [43,44] 1978, 79

May & Oster [42] 1980

$\boxed{\text{May 1976}}$

Explorat° diagramme phase

+ définit° α & δ

+ fenêtres périodiqu

Li & Yorke [2] 1975

Sharkovskii ordering.

Sharkovskii [38] 1964

Myrberg [37] 1958

Metropolis [40] 1973

1 2 4 … 2^n …

$\times 2$

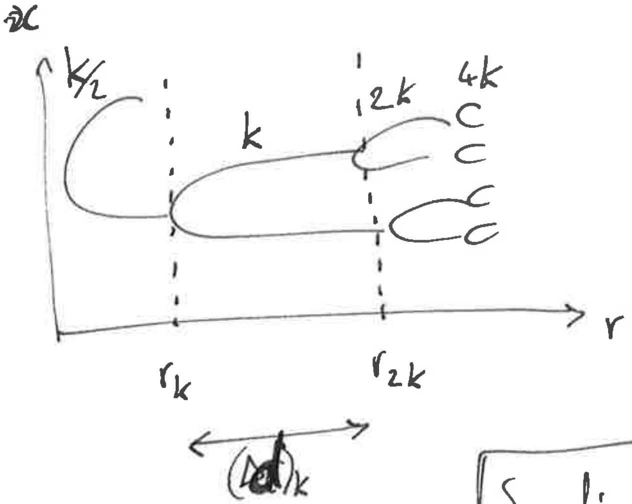
$\times 2$

impairs … 7 5 3 $\times 2$

IV a

$f_r(x) = r\,x(1-x)$

$f_r^{(k)}(x) = [f_r \circ \dots \circ f_r](x)$



$$\boxed{\delta = \lim_{k\to\infty} \frac{d_k}{d_{2k}}}$$

but : calculer δ

entre r_k & r_{2k} : $\exists X_k$ tq $f_r^{(k)}(X_k) = X_k$ (stable)

$$g_k = f'\Big|_{X_k} = \frac{df}{dx}\Big|_{X_k}$$

⚠ $g_k(r_k) = 1$ (stabilisation)

$g_k(r_{2k}) = -1$ (subharmonique) $\quad g_{2k}=1$

$|g_k(r\in]r_k, r_{2k}[)| < 1$ (stable) $\quad g_{2k}=-1$

but calculer d_k

Si on écrit $$g_k(r_k+\delta r) = 1 - q\,\delta r + \cancel{O(\delta r^2)}$$ NSM

$$g_k(r_{2k}) = -1 = 1 - q\,d_k \Rightarrow q\,d_k = 2$$

$$g_k(r_{4k}) = g_k^* = 1 - q(d_k + d_{2k}) = 1 - 2 - q\,d_{2k}$$

$$\Rightarrow q\,d_{2k} = -(g_k^* + 1)$$

$$\Rightarrow \delta = \frac{q\,d_k}{q\,d_{2k}} = \frac{-2}{1+g_k^*}$$

* but : trouver g_k^*

$$g_k = \left.\frac{\partial f_r}{\partial x}\right|_{x=X_k}$$

$$f_r^{(k)}(X_k+\xi) = X_k + g_k \xi + h_k \xi^2 + \dots \qquad h_k = \frac{1}{2}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x=X_k}$$

$$f_r^{(2k)}(X_k+\xi) = f_r^{(k)}\left(X_k + g_k\xi + h_k\xi^2 + O(\xi^3)\right)$$

$$= X_k + g_k\left[g_k\xi + h_k\xi^2 + O(\xi^3)\right] + h_k\left[g_k\xi + h_k\xi^2 + O(\xi^3)\right]^2 + O(\xi^3)$$

$$= X_k + (g_k)^2\xi + \overbrace{g_k h_k\left[1+g_k\right]}^{B}\xi^2 + \frac{1}{3}C\xi^3 + \dots$$

Par exemple : à $r \simeq r_{2k}$, on a $g_k \simeq -1$ et donc $g_k^2 \simeq +1$ et $B \simeq 0$

↳ cf case !

$(g_k)^2 = g_{2k} = +1$

$$X_{2k} = X_k + \xi^* \Rightarrow \xi^* = (g_k)^2\xi^* + B\xi^{*2} + \frac{1}{3}C\xi^{*3} \Rightarrow \boxed{\frac{1}{3}C\xi^{*2} = 1-(g_k)^2 - B\xi^*}$$

$$g_{2k} = (g_k)^2 + B\xi^* + C\xi^{*2} = 3 - 2(g_k)^2 - \underbrace{2B\xi^*}_{\text{we neglect bcz } \xi^* \ll 1}$$

or at r_{2k} : $g_{2k} = -1 \Rightarrow (g_k^*)^2 = 2 \Rightarrow \boxed{g_k^* = \sqrt{2}}$ (<0)

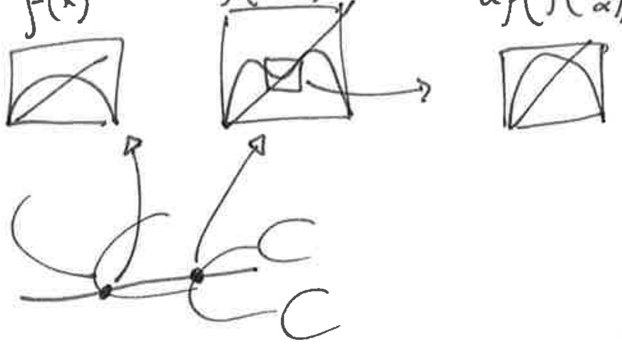
$$\Rightarrow \delta = \frac{-2}{1-\sqrt{2}} = 2(1+\sqrt{2})$$ ✓

$\simeq 4.828$ 3%

Renormalisat° à la mano

IVa

$f(x)$ $\quad$ $f(f(x))$ $\quad$ $\alpha f\left(f\left(\frac{x}{\alpha}\right)\right) = T[f]$



$$k \to 2k$$

$$f \to T[f]$$

à la fin on arrive à $g(x)$ tq. $g = Tg$

$$f(x) = 1 - c\,x^2$$

$$Tf(x) = \alpha\left[1 - c\left(1 - c\left(\frac{x}{\alpha}\right)^2\right)^2\right]$$

$$= (1-c)\,\alpha + \frac{2c^2}{\alpha}x^2 - \frac{c^3}{\alpha^3}x^4 \Rightarrow \alpha = \frac{1}{1-c}$$

$$\boxed{\begin{array}{l} \alpha = 2.503 \\ \delta = 4.669 \end{array}}$$

2 approches : $Tf(x) \simeq f(x)$ $\qquad$ $Tf(1) \simeq f(1)$

$$\bar{c} = \underbrace{-2c^2(1-c)}_{R(c)}$$

$$c_\infty = \frac{1 \pm \sqrt{3}}{2} \simeq 1.366$$

$(c_\infty > 1$ bcz $\alpha < 0)$

$$\alpha_\infty = -1 - \sqrt{3} \simeq -2.732$$

$$\bar{c} = -2c^2(1-c) + c^3(1-c)^3$$

$$c_\infty \simeq 1.402$$

$$\alpha_\infty = -2.48$$

V b

$c_{k+1} = R(c_k) \qquad R(c) = -2c^2(1-c) \qquad \left(R(c) = -2c^2(1-c) + c^3(1-c)^3\right)$

évolut° du paramètre : on cherche

$$\delta = \lim_{\infty} \frac{c_{k+1}-c_k}{c_k - c_{k-1}}$$

$$= \lim_{\infty} \frac{c_{k+1} - c_\infty - [c_k - c_\infty]}{c_k - c_\infty - [c_{k-1} - c_\infty]}$$

$$c_{k+1} - c_\infty = R(c_k) - c_\infty = R(c_\infty + \overbrace{c_k - c_\infty}^{\text{small}}) - c_\infty$$

$$= R'(c_\infty)[c_k - c_\infty]$$

$$= \lim_{\infty} \frac{[R'(c_\infty) - 1][c_k - c_\infty]}{[R'(c_\infty) - 1][c_{k-1} - c_\infty]}$$

$$= R'(c_\infty)$$

$$c_{k+1} = R(c_k)$$

$$= R(c_\infty + \underbrace{c_k - c_\infty}_{\ll 1})$$

$$= \underbrace{R(c_\infty)}_{c_\infty} + R'(c_\infty)[c_k - c_\infty]$$

$$\frac{c_{k+1} - c_\infty}{c_k - c_\infty} = R'(c_\infty)$$

$$\delta = \lim \frac{c_{k+1} - c_k}{c_k - c_{k-1}} = \frac{c_{k+1} - c_\infty - (c_k - c_\infty)}{c_k - c_\infty - (c_{k-1} - c_\infty)} = \frac{\cancel{(R'-1)}(c_k - c_\infty)}{\cancel{(R'-1)}(c_{k-1} - c_\infty)} = R'(c_\infty)$$

$R' = -4c + 6c^2$

$\delta = 4 + \sqrt{3}$

$\simeq 5.732$

27%

$R' = -4c + 6c^2 + 3c^2(1-c)^2[1-2c]$

$\delta \simeq 4.467$

4%