Hydrodynamical transport in a low temperature Fermi gas

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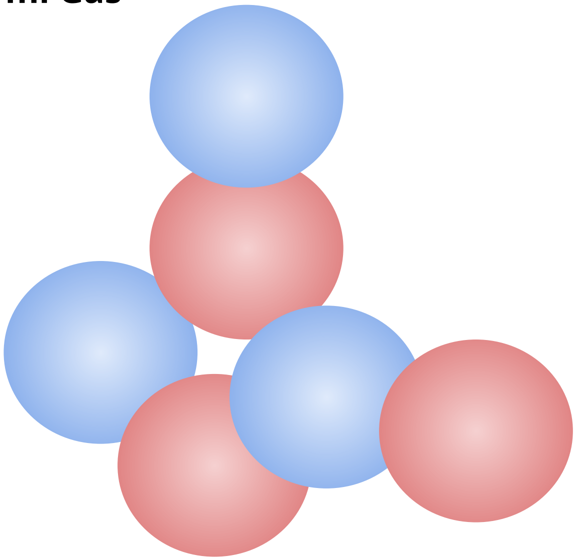
Les Gustins - summer 2026

Ultracold atoms and Fermi gas

Dilute cold gas with contact interaction tuned by a single parameter : the s-wave scattering length ***a***

Two-component Fermi Gas



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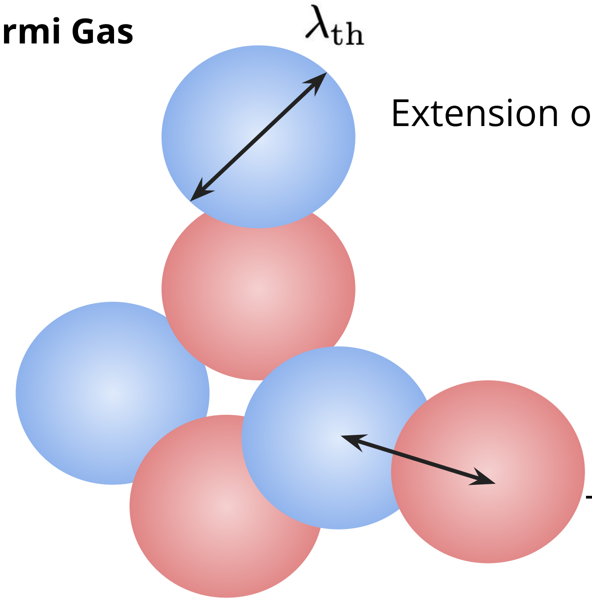

Extension of the wave function $\lambda_{\mathrm{th}} = \sqrt{\dfrac{2\pi\hbar^2}{mk_{\mathrm{B}}T}}$

Typical interparticle distance $\rho_0^{-1/3}$

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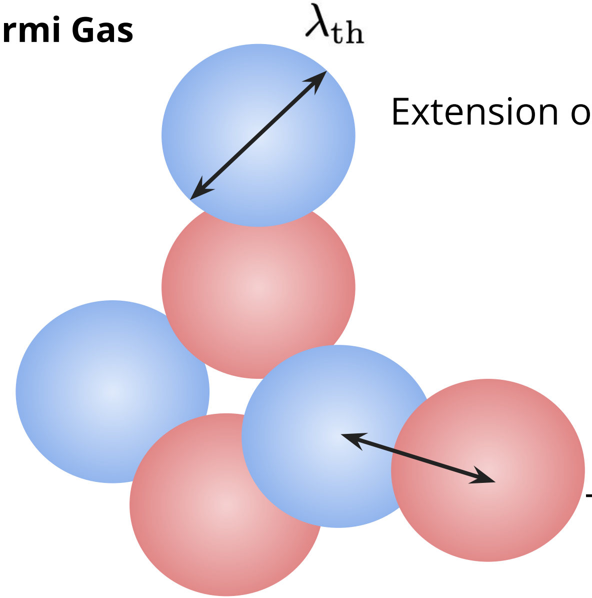

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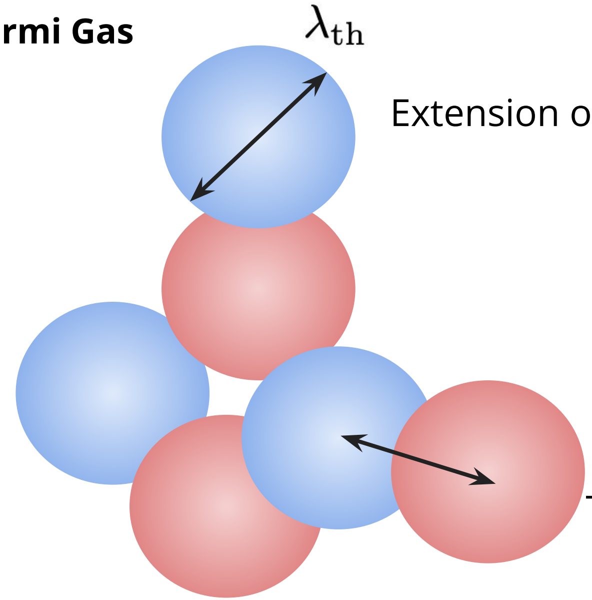

Extension of the wave function $\lambda_{\mathrm{th}} = \sqrt{\dfrac{2\pi\hbar^2}{m k_{\mathrm{B}} T}}$

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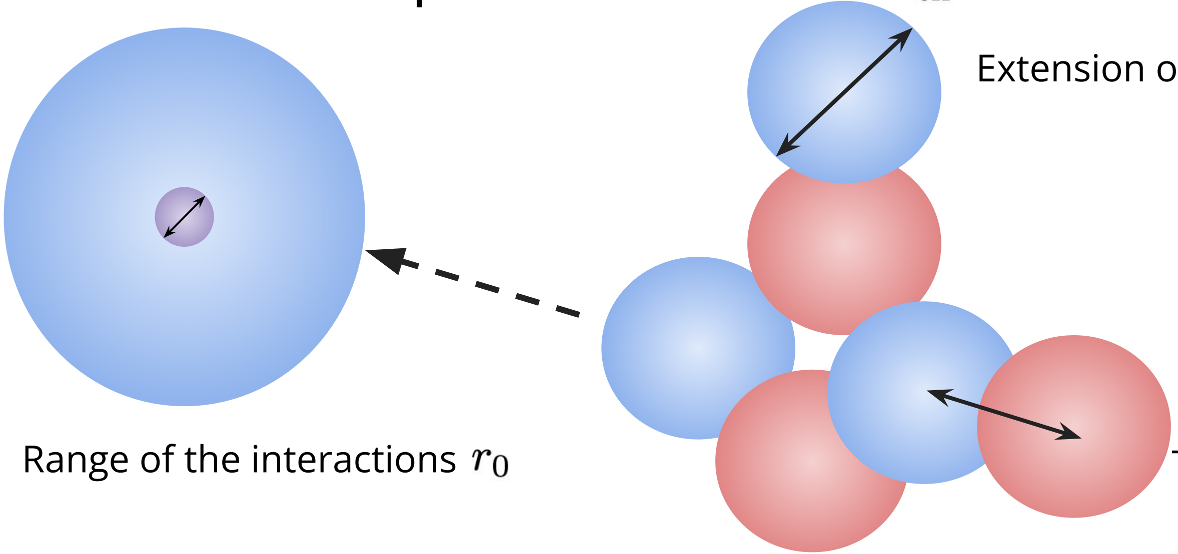

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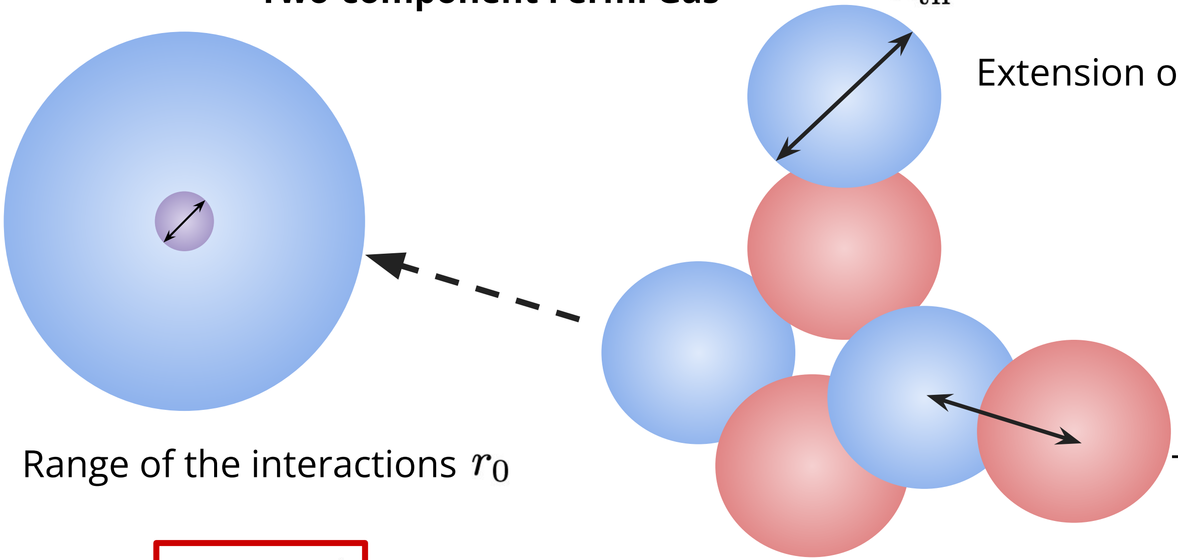

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$$\boxed{\begin{array}{l} r_0 \ll \rho_0^{-1/3} \\ r_0 \ll \lambda_{\mathrm{th}} \end{array}}$$

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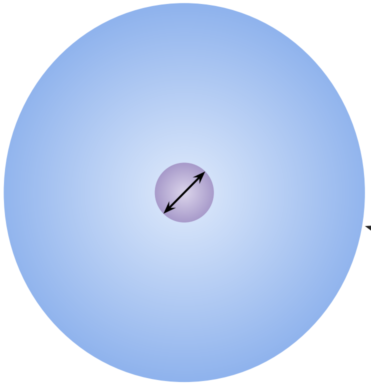
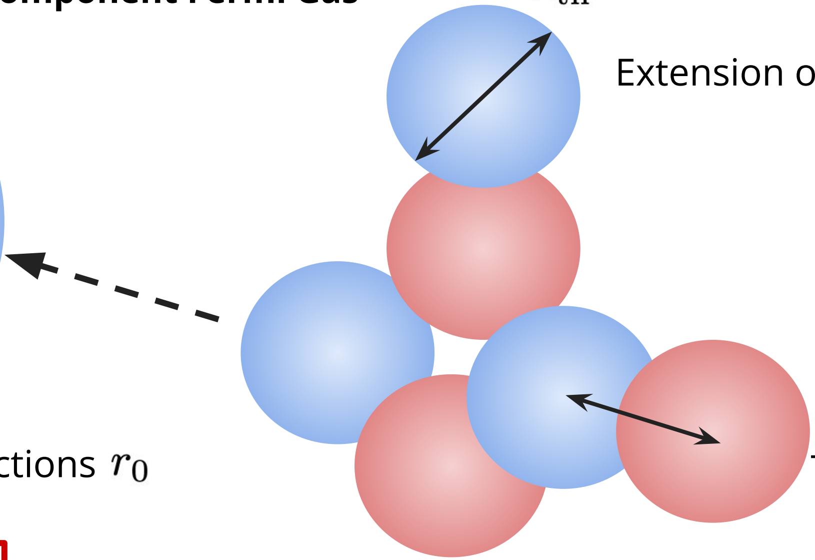
$\lambda_{\rm th}$





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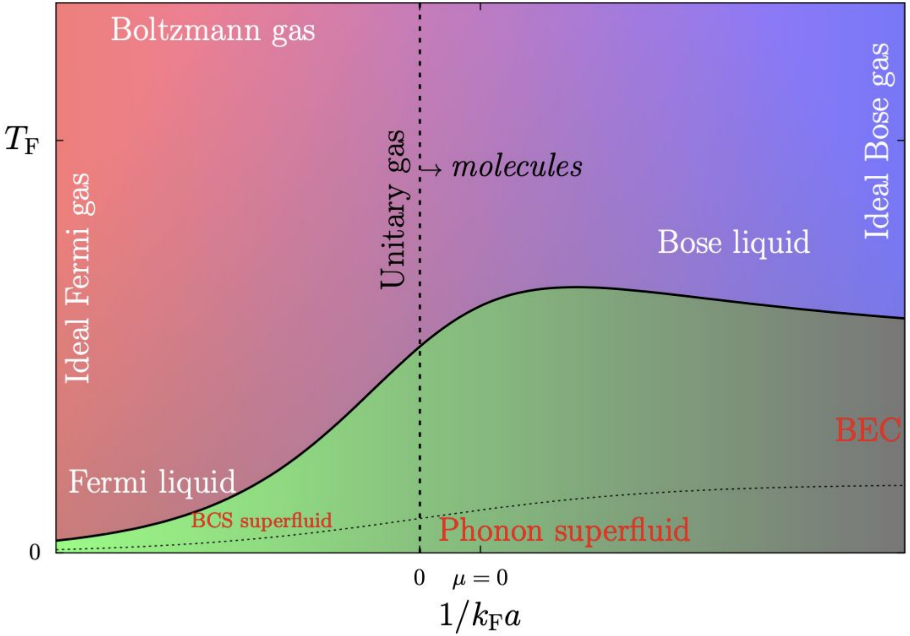
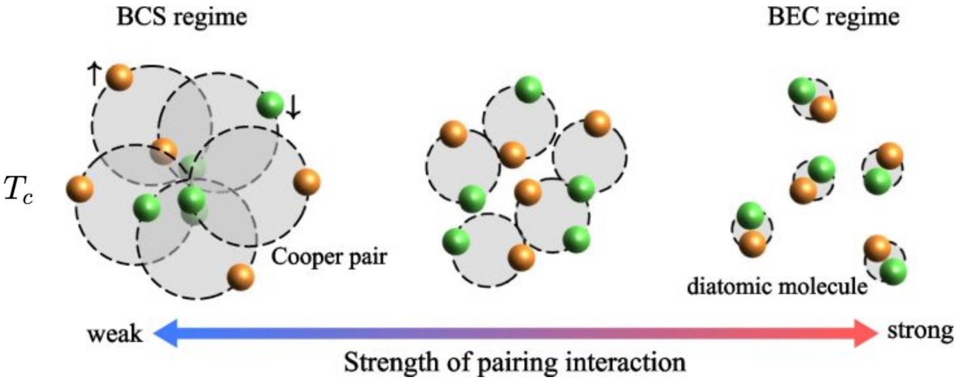
Range of the interactions r_0

Typical interparticle distance $\rho_0^{-1/3}$

$$\boxed{\begin{array}{l} r_0 \ll \rho_0^{-1/3} \\ r_0 \ll \lambda_{\rm th} \end{array}} \longrightarrow \boxed{V_{\rm VdW}(\mathbf{r}) \to V(\mathbf{r}) = g_0(k_{\rm F}a)\ \delta(\mathbf{r})}$$

Ultracold atom and Fermi gas





<u>Phase diagram of the Fermi gas with contact interactions</u>

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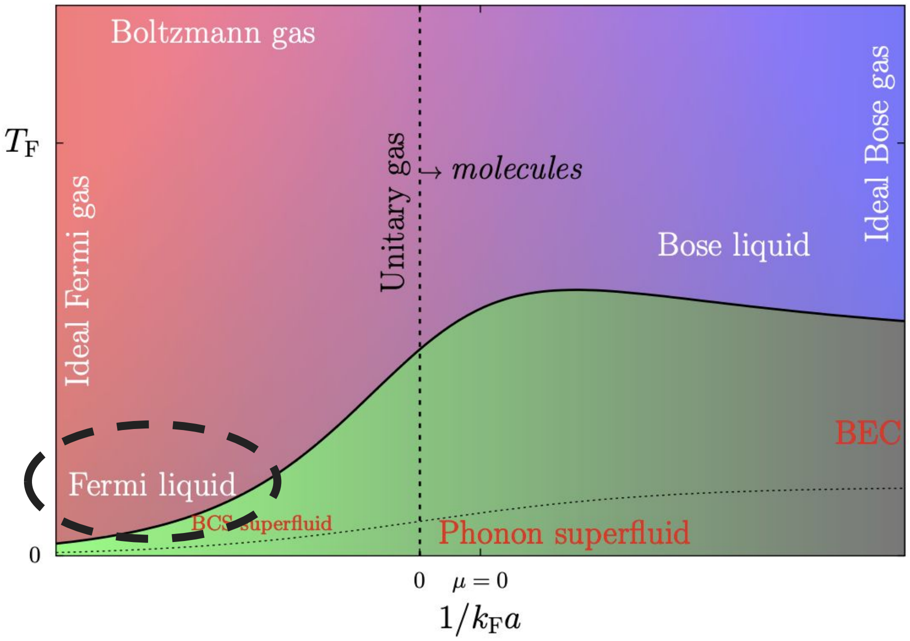
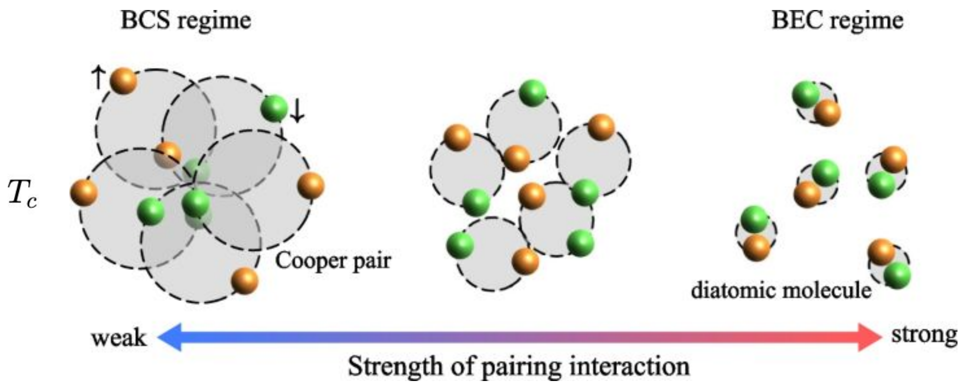
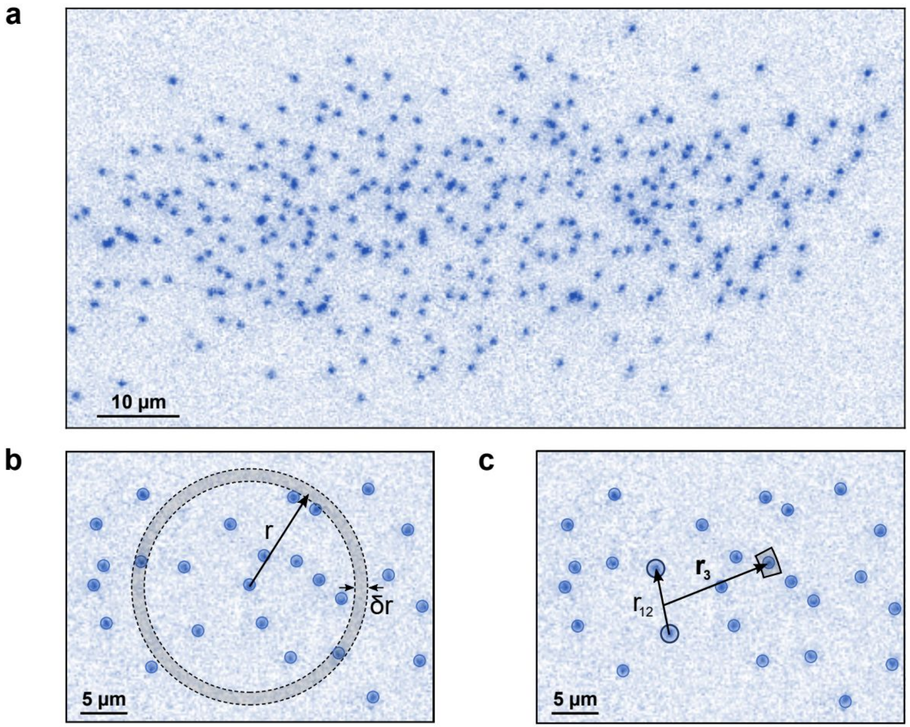




Phase diagram of the Fermi gas with contact interactions

What can actually be measured?



<u>But also :</u>

- Equation of state in different regime
- Signature of Superfluid phase transition

Tim de Jongh,et al. Quantum gas microscopy of fermions in the continuum. Phys. Rev. Lett., 134:183403, May 2025.

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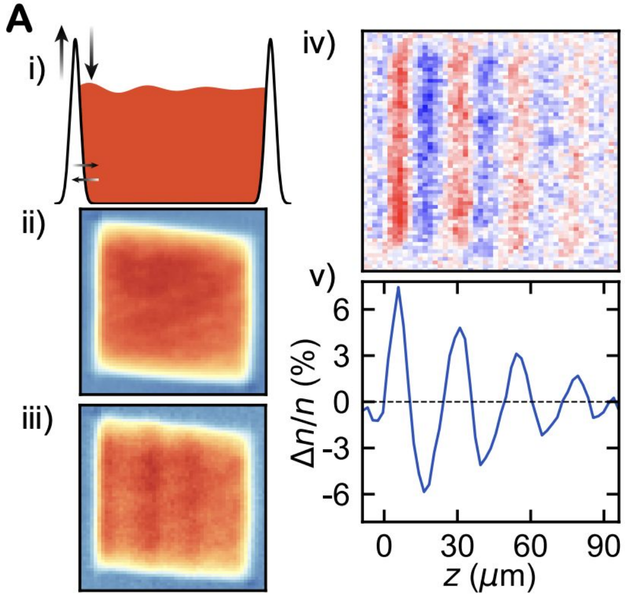


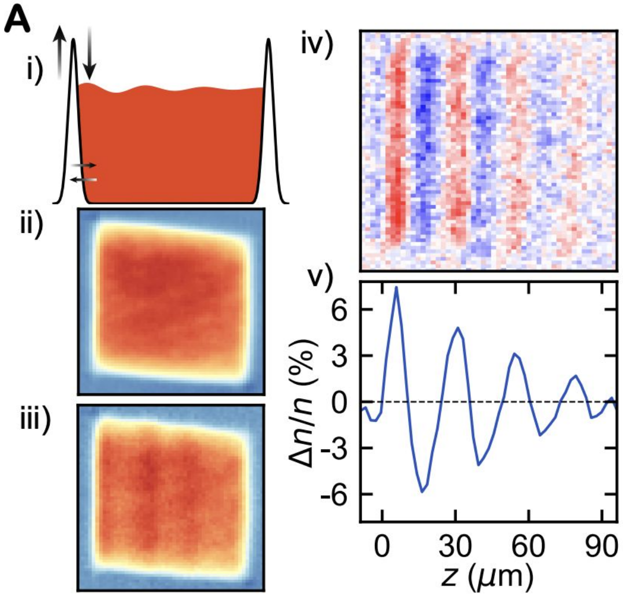
Parth B. Patel et al. ,Universal sound diffusion in a strongly interacting Fermi gas.Science ***370***,*1222-1226(2020)*

Transport coefficients : the main aim of that presentation

$$\eta = \eta_0[1 + \alpha(k_F a)]$$

Similar to the LHY expansion

What can actually be measured?



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Transport coefficients : the main aim of that presentation

$$\eta = \eta_0 [1 + \alpha(k_F a)]$$

Only approximations with errors of up to 20%

Not calculated yet

How can we capture the complexity of the microscopic model in order to reduce it to its principal degrees of freedom?

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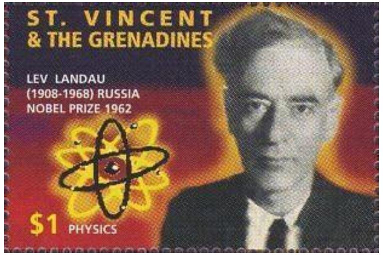
Landau Fermi Liquid Theory (FLT)

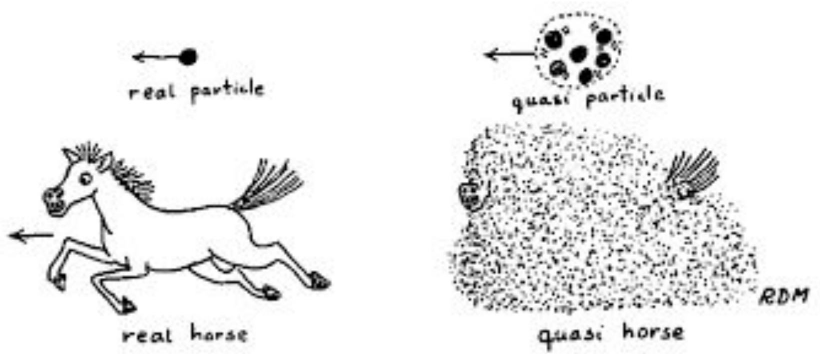
Landau's FLT :

Developed in the 60's for the ^{3}He , latter used to describe electrons in metal and other fermionic systems



Lev Landau

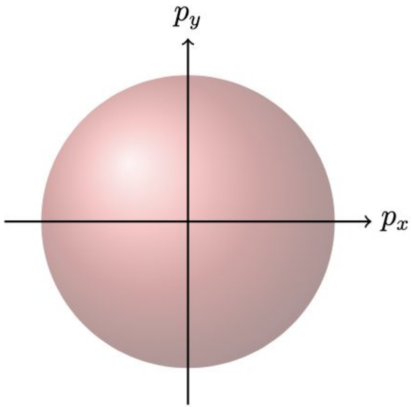
This theory lies onto the concept of **quasiparticles** : effective description of the system



More precisely, the system is viewed as an effective **dilute gas of quasiparticles excitations** → <u>From a strong correlated gas to a dilute gas with few body interactions</u>

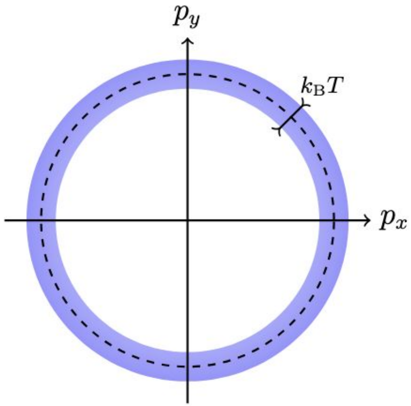
Landau’s FLT :

The main idea is that a quasiparticle gas is very similar to an ideal particle gas, provided that the interactions within it are "switched on" **adiabatically**



Adiabatic switch on of interactions

$$T_c \leq T \ll T_F$$



Fermi Sea of an ideal gas

Quasiparticles living near the Fermi surface

The system is thus described by a **semiclassical distribution of quasiparticles**

$$n_\sigma(\mathbf{p}, \mathbf{r}, t) = n^0_\sigma(\mathbf{p}) + \delta n_\sigma(\mathbf{p})$$

Landau’s FLT : The toolbox

<u>A semi-classical action on the quasiparticle density fluctuations</u>

$$E[\delta n_\sigma] = E_0 + \sum_{\mathbf{p}\sigma} \epsilon_{\mathbf{p}\sigma}\ \delta n_\sigma(\mathbf{p}) + \frac{1}{2L^3} \sum_{\mathbf{p},\mathbf{p}',\sigma,\sigma'} f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')\ \delta n_\sigma(\mathbf{p})\delta n_{\sigma'}(\mathbf{p}')$$

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Dispersion relation :

$$\epsilon_{\mathbf{p}\sigma} - \mu = \frac{p_{\mathrm{F}}}{m^*}(p - p_{\mathrm{F}})$$

Quasiparticles effective mass m*

Landau's FLT : The toolbox

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Interaction functions :

Angular dependence only →we can do a Legendre projection

$$f_{\sigma\sigma'}(\theta) \to F_l$$ **Landau coefficients**

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Landau coefficients

Static parameters :

$$\frac{m^*}{m} = 1 + \frac{F_1^s}{3}$$

Compressibility : $\chi^{-1} = \frac{p_\mathrm{F}^2}{mm^*}(1 + F_0^s)$

Landau’s FLT : Why is it not enough ?

- The FLT doesn’t include complete **collision amplitude** which are necessary for studying the non-equilibrium dynamics and relaxation of quasiparticles

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$$\Gamma_{\mathbf{p}} = \frac{2\pi}{L^6} \sum_{\mathbf{p}_2 \mathbf{p}_3 \mathbf{p}_4} |\mathcal{A}(\mathbf{p}, \mathbf{p}_2 | \mathbf{p}_3, \mathbf{p}_4)|^2 \; \delta(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \; \delta(\epsilon_{\mathbf{p}} + \epsilon_{\mathbf{p}_2} - \epsilon_{\mathbf{p}_3} - \epsilon_{\mathbf{p}_4})$$
$$\times \, n_{\mathrm{eq}}(\mathbf{p}_2)(1 - n_{\mathrm{eq}}(\mathbf{p}_3)(1 - n_{\mathrm{eq}}(\mathbf{p}_4)))$$

$$n_{\mathrm{eq}}(\mathbf{p}) = \frac{1}{1 + \mathrm{e}^{(\epsilon_{\mathbf{p}} - \mu)/T}}$$

Fermi-Dirac Distribution

<u>Quasiparticle lifetime :</u> a four momenta collision amplitude appear → depends on at least **two angles**

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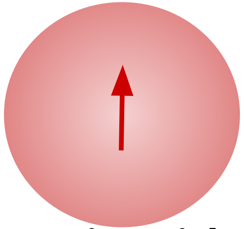
Fermi-Dirac Distribution

Quasiparticle lifetime : a four momenta collision amplitude appear

- The theory cannot describe what happens at the **normal-to-superfluid transition** : no prediction of the **critical temperature**, absence of the pairing amplitudes

Quasiparticles as dressed particles:



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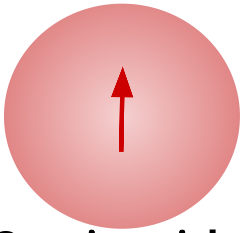
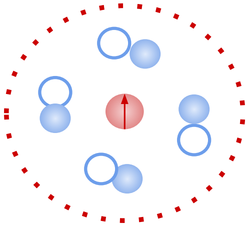


Quasiparticle

Particle dressed by the interactions with
“ high-energy ” particles



Pair of
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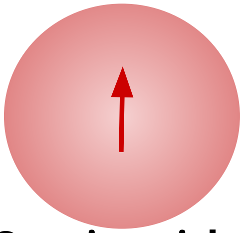

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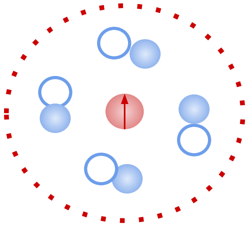

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$$\hat{\gamma}_{\mathbf{p}\uparrow} = \mathrm{e}^{\hat{S}}\hat{a}_{\mathbf{p}\uparrow}\mathrm{e}^{-\hat{S}} = \hat{a}_{\mathbf{p}\uparrow} + \sum_{\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4} \mathcal{C}_V(\mathbf{p},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4)\; \hat{a}^{\dagger}_{\mathbf{p}_2\downarrow}\hat{a}_{\mathbf{p}_3\downarrow}\hat{a}_{\mathbf{p}_4\uparrow} + \cdots$$

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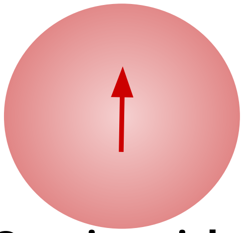

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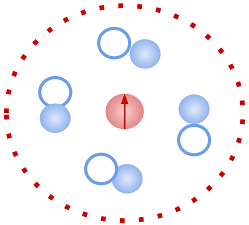

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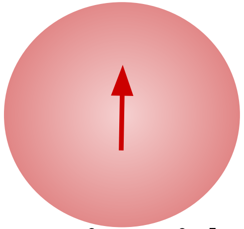

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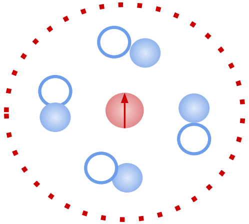

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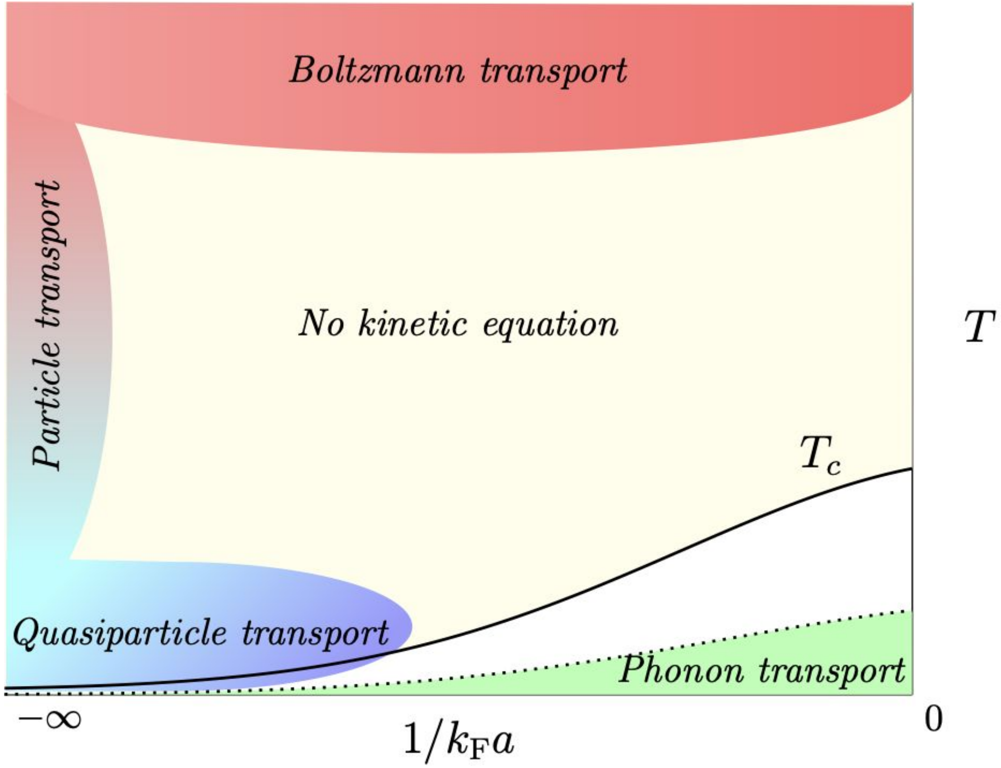
$$\boxed{\hat{H} = E_0 + \sum_{\mathbf{p}\sigma} \epsilon_{\mathbf{p}\sigma}\ \delta\hat{n}_{\mathbf{p}\sigma} + \frac{1}{2L^3} \sum_{\substack{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4 \\ \sigma,\sigma'}} \mathcal{A}_{\sigma\sigma'}(\mathbf{p}_1,\mathbf{p}_2|\mathbf{p}_3,\mathbf{p}_4)\ \delta(\hat{\gamma}^\dagger_{\mathbf{p}_1\sigma}\hat{\gamma}_{\mathbf{p}_4\sigma})\delta(\hat{\gamma}^\dagger_{\mathbf{p}_2\sigma'}\hat{\gamma}_{\mathbf{p}_3\sigma'})}$$

Effective Hamiltonian linearized about the Fermi sea

Transport and hydrodynamics of a Fermi gas

When can we talk about transport in a Fermi gas?



In order to derive a transport equation and break the BBGKY hierarchy, one needs a clear **time scale separation**

Molecular chaos hypothesis

We can apply standard quantum kinetic theory and derive a Boltzmann equation for the distribution $n_\sigma(\mathbf{p})$

A transport equation for the quasiparticles :

$$i\partial_t n_\sigma(\mathbf{p}) - \frac{\mathbf{p}\cdot\mathbf{q}}{m^*} n_\sigma(\mathbf{p}) + \frac{\partial n_{\mathrm{eq}}}{\partial \epsilon}\frac{\mathbf{p}\cdot\mathbf{q}}{m^*}\left(U_{\mathrm{ext}} + \frac{1}{L^3}\sum_{\mathbf{p}'} f_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')n_{\sigma'}(\mathbf{p}')\right) = iI_\sigma(\mathbf{p})$$

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Ballistic motion

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Ballistic motion

External and mean-field forces

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Ballistic motion

External and mean-field forces

$$\omega_0 = v_{\mathrm{F}} q$$

Typical excitation frequency

A transport equation for the quasiparticles :

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Ballistic motion

External and mean-field forces

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$$1/\tau \propto T^2 \mathcal{A}^2$$

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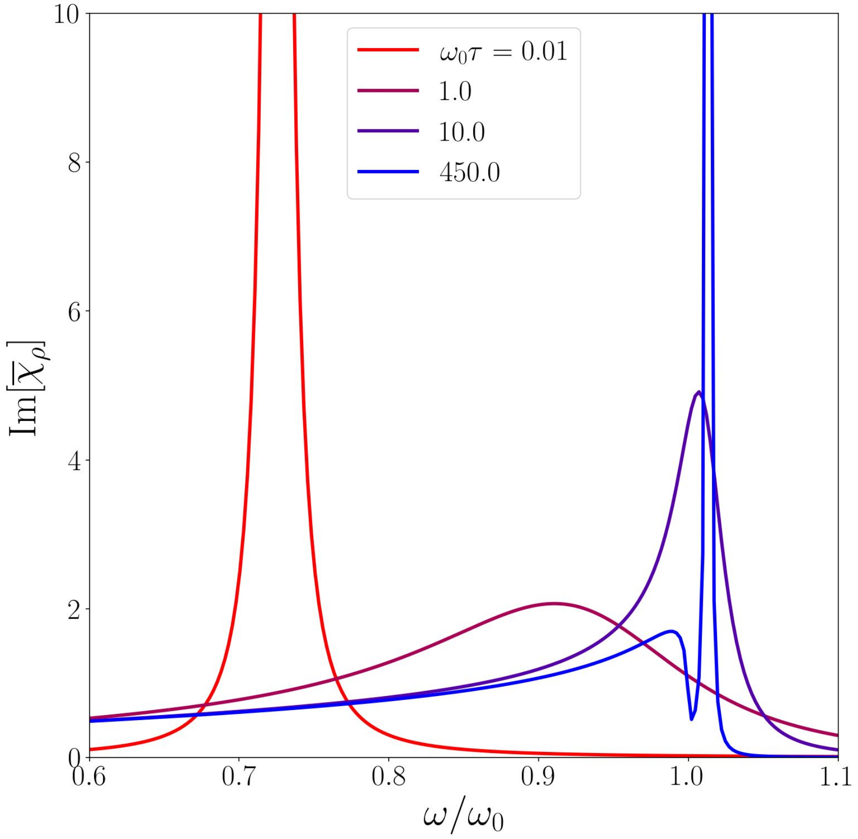
From the collisionless to the hydrodynamics regime :



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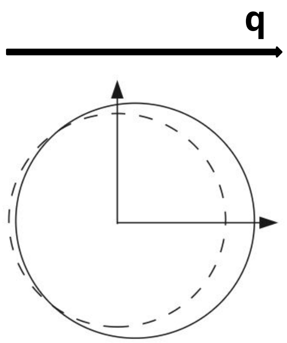
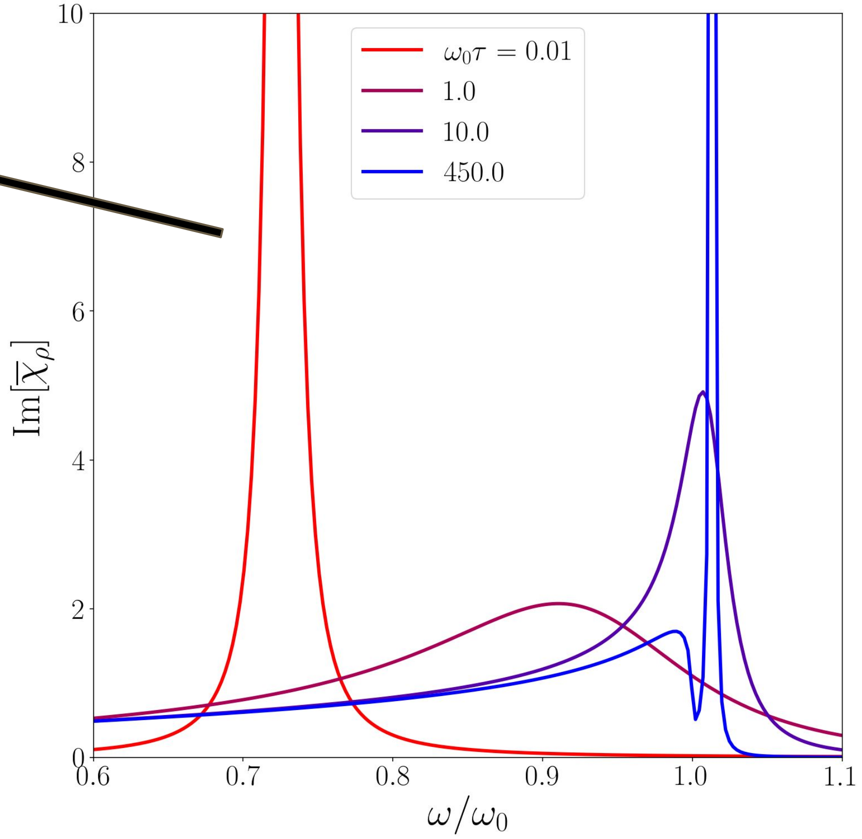


Translation of the
Fermi surface



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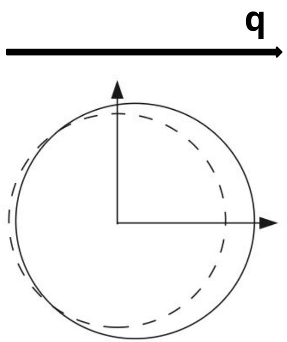

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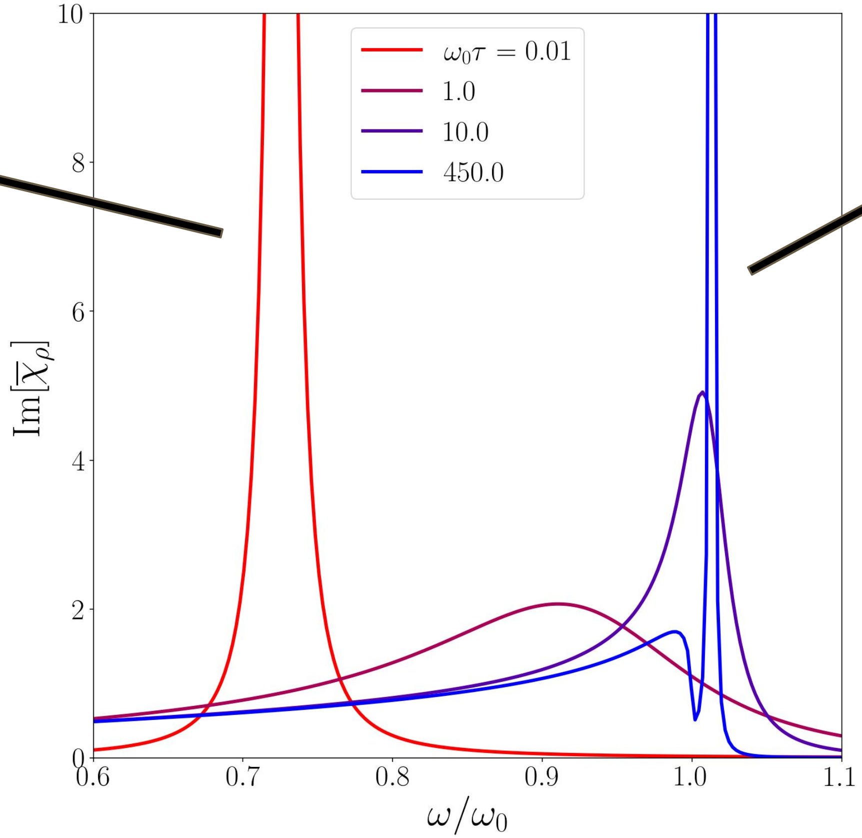
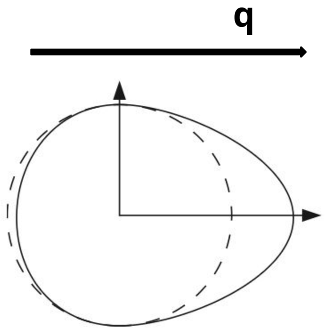


Zero sound



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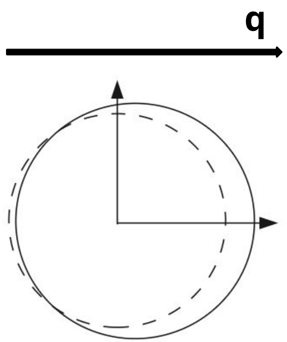

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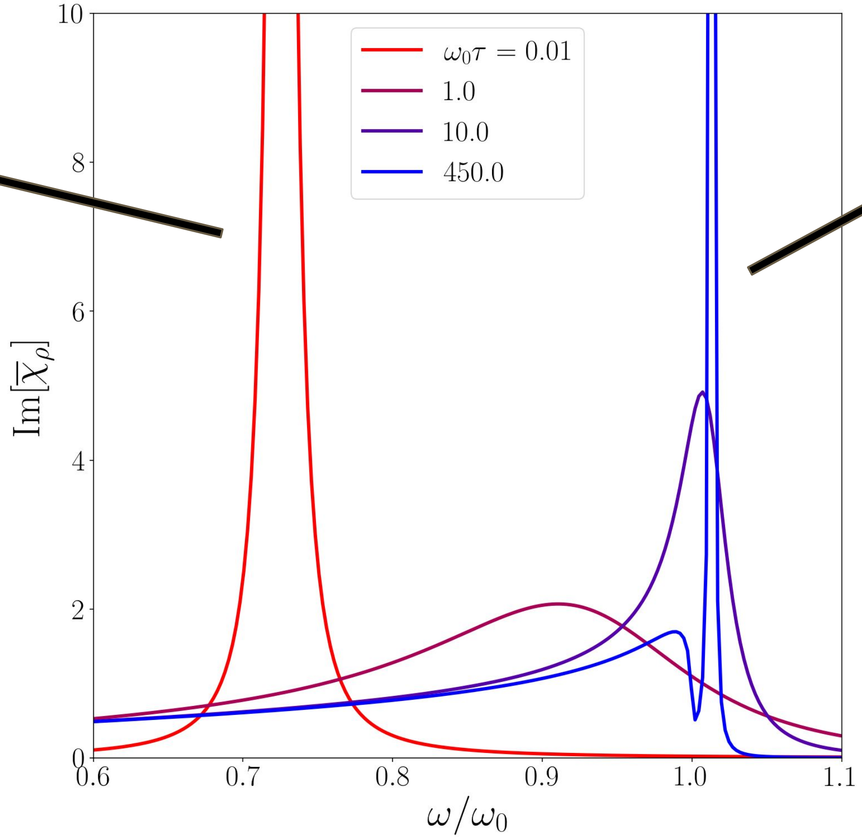

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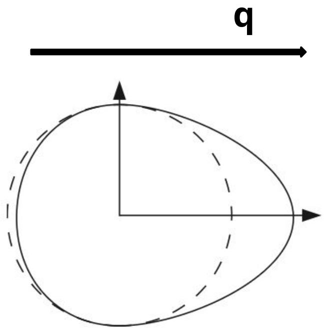

Deformation of the Fermi surface

How to derive Navier-Stokes equations ?

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$$\sum_{\mathbf{p}} I_\sigma(\mathbf{p}) = 0$$

$$\rho = \frac{1}{L^3}\sum_{\mathbf{p}\sigma} n_{\mathbf{p}\sigma}$$ **Density conservation**

$$P = \frac{1}{L^3}\sum_{\mathbf{p}}(n_\uparrow(\mathbf{p}) - n_\downarrow(\mathbf{p}))$$ **Polarization conservation**

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$$\sum_{\mathbf{p}} \mathbf{p}\; I_\sigma(\mathbf{p}) = 0 \longrightarrow \rho_0 \mathbf{v} = \frac{1}{L^3}\sum_{\mathbf{p}\sigma} \frac{\mathbf{p}}{m}\; n_\sigma(\mathbf{p})$$

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$$\sum_{\mathbf{p}} p^2\ I_\sigma(\mathbf{p}) = 0 \longrightarrow e = \frac{1}{L^3}\sum_{\mathbf{p}\sigma} \left(\frac{p^2}{2m^*} - \mu\right) n_{\mathbf{p}\sigma}$$

Energy conservation

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Let us start with the **continuity equation** :

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We recognize the total density and the divergence of the total velocity :

$$\boxed{\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v_{||}}{\partial z} = 0}$$

Continuity equation

How to derive Navier-Stokes equations ?

Now we look at the equation for the total velocity :

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The total velocity is coupled to :

- the total density $\rho = \frac{1}{L^3}\sum_{\mathbf{p}\sigma} n_{\mathbf{p}\sigma}$
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l =2 Legendre Polynomial

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$$\Longrightarrow \quad m\rho_0\, \partial_t v_{||} + \frac{1}{\rho_0\chi}\frac{\partial\rho}{\partial z} - \rho_0 F_{\text{ext}} + \frac{\partial\Pi}{\partial z} = 0$$

Stress tensor

Hydrodynamics equation for a Fermi Liquid

Using the same method for the other conserved quantities, we finally obtain the following set of hydrodynamic equations :

- Continuity equation :

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v_{||}}{\partial z} = 0$$

- Heat equation :

$$c_V \frac{\partial e}{\partial t} = \kappa \frac{\partial^2 e}{\partial z^2}$$

- Spin diffusion equation :

$$\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial z^2} + \chi_{\mathrm{s}} D \frac{\partial F_{\mathrm{ext}}}{\partial z}$$

- Navier-Stokes equation :

$$m\rho_0 \frac{\partial v_{||}}{\partial t} + \frac{1}{\rho_0 \chi} \frac{\partial \rho}{\partial z} - \rho_0 F_{\mathrm{ext}} = \frac{4}{3}\eta \frac{\partial^2 v_{||}}{\partial z^2}$$

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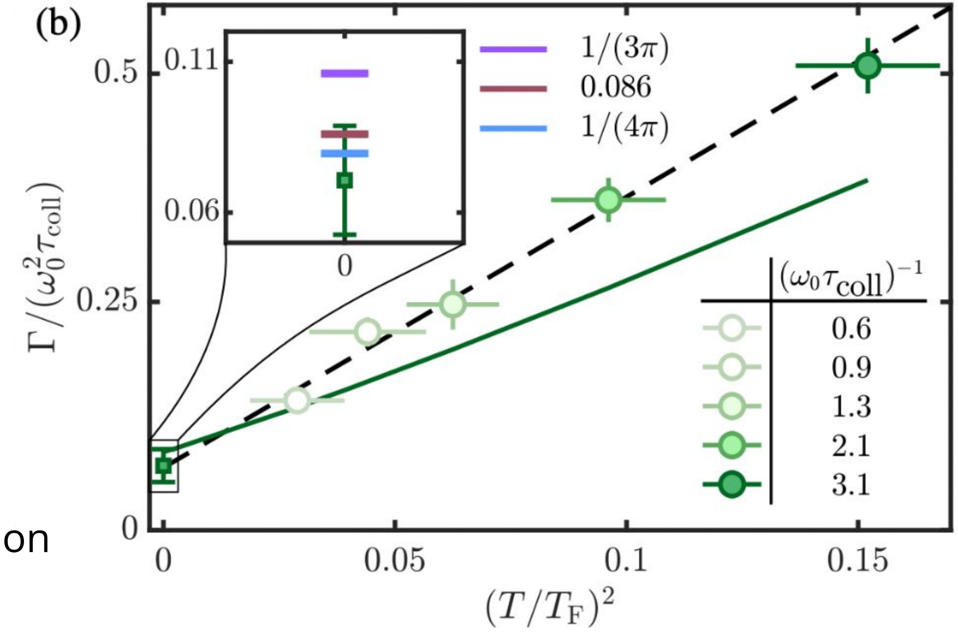
Transport coefficients of a low-temperature Fermi gas

Comparison with experiments

Shear viscosity

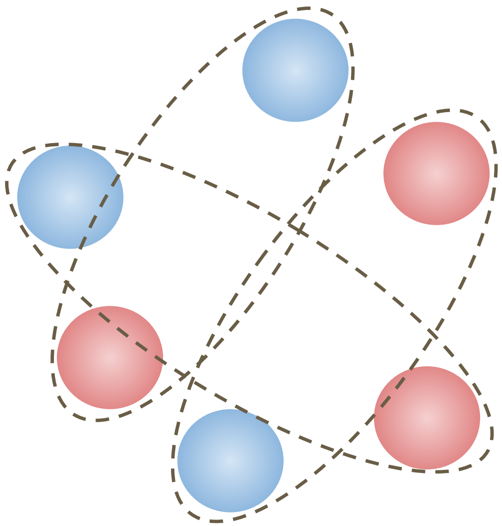
$$\text{Extrapolation to } T = 0 \text{ value :}$$
$$\overline{\eta}^{\text{exp}}(k_{\text{F}}a = -0.67) = 0.070$$
$$\overline{\eta}^{(1)}(k_{\text{F}}a = -0.67) = 0.086$$
$$\overline{\eta}^{(2)}(k_{\text{F}}a = -0.67) = 0.071$$

Error bars due to the temperature extrapolation



S. Huang,et al , Emergence of Sound in a Tunable Fermi Fluid, Phys. Rev. X ***15****, 011074 (2025).*

Conclusion



And now ?

- **Two fluids hydrodynamics** in the Superfluid phase

$$\rho = \rho_n + \rho_s \qquad \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s$$

A rich physics of transport in the Superfluid

- **Phase transition :** We have already shown that the well-known GMB correction at the critical temperature is naturally present in our formalism

Conclusion

Appendix

Quasi degenerate perturbation theory :

Consider a 3D gas of particles described by the following Hamiltonian : $\hat{H} = \hat{H}_0 + \hat{V}$

where the kinetic energy is written as $$\hat{H}_0 = \sum_{\mathbf{p}\sigma} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}\sigma} \hat{a}_{\mathbf{p}\sigma} \quad \Rightarrow \left\{ E_n^0, |n\rangle_0 \right\}$$

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We see the quasiparticles as **perturbed particle states** $\rightarrow$ <u>Perturbation theory</u>

$$|\{n_{\mathbf{p}\sigma}\}\rangle = |\{n_{\mathbf{p}\sigma}\}\rangle_0 + |\{n_{\mathbf{p}\sigma}\}\rangle_1 + |\{n_{\mathbf{p}\sigma}\}\rangle_2 + \dots \quad \Longrightarrow \quad \boxed{|n\rangle = \mathrm{e}^{\hat{S}}|n\rangle_0}$$

Unitary transformation

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Unitary transformation

But since the spectrum is a continuum, we cannot apply the standard perturbation method

C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Processus d'interaction entre photons et atomes

J. H. Van Vleck, On σ-Type Doubling and Electron Spin in the Spectra of Diatomic Molecules, Phys. Rev. ***33****, 467 (1929)*

J. R. Schrieffer and P. A. Wolff, Relation between the Anderson and Kondo Hamiltonians, Phys. Rev. ***149****, 491 (1966)*

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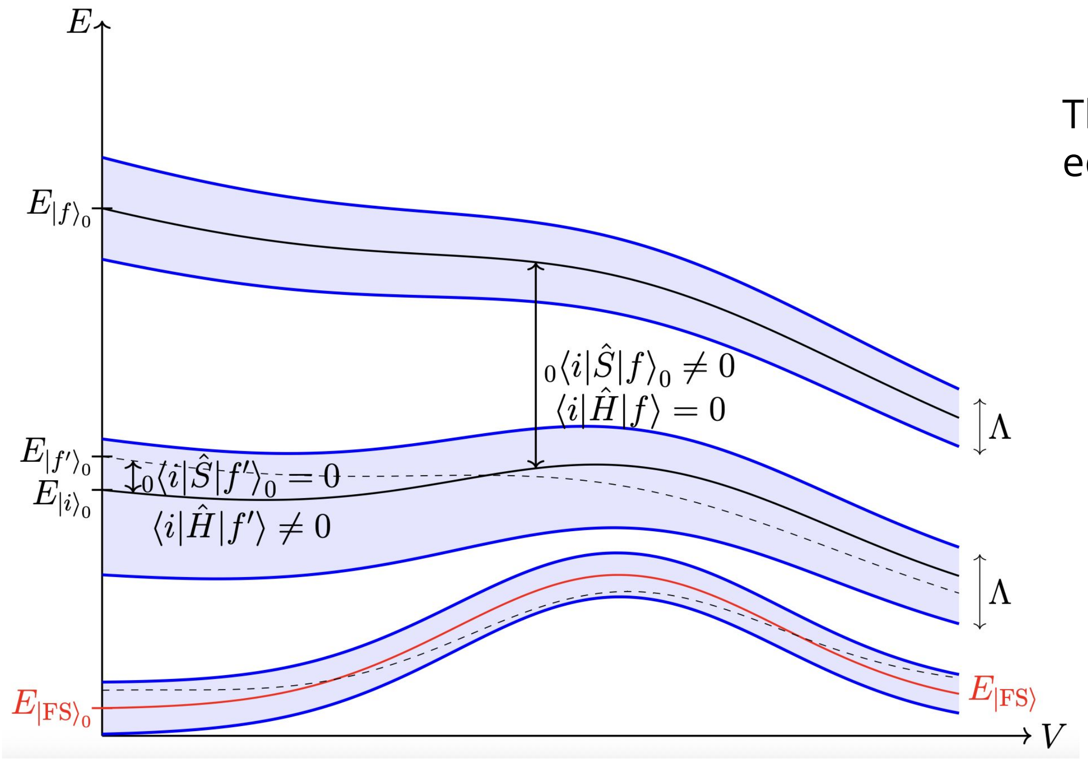

Non-crossing criterion

The concept of **non-crossing** is the equivalent of adiabadicity:

$$\boxed{|E_i^0 - E_f^0| \ll \Lambda \Leftrightarrow |E_i - E_f| \ll \Lambda}$$

The energy structures of the perturbed and unperturbed systems are similar

Energy cutoff

$$\Gamma_{\mathrm{typ}},\ \omega_{\mathrm{ext}} \ll \Lambda \ll \epsilon_{\mathrm{F}}$$

Quasi degenerate perturbation theory :

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We can also directly transform the operators, especially we can define **quasiparticles creation/annihilation operators :**

$$\hat{\gamma}_{\mathbf{p}\uparrow} = \mathrm{e}^{\hat{S}} \hat{a}_{\mathbf{p}\uparrow} \mathrm{e}^{-\hat{S}} = \hat{a}_{\mathbf{p}\uparrow} + \sum_{\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4} \mathcal{C}_V(\mathbf{p},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4)\, \hat{a}^\dagger_{\mathbf{p}_2\downarrow} \hat{a}_{\mathbf{p}_3\downarrow} \hat{a}_{\mathbf{p}_4\uparrow} + \cdots$$

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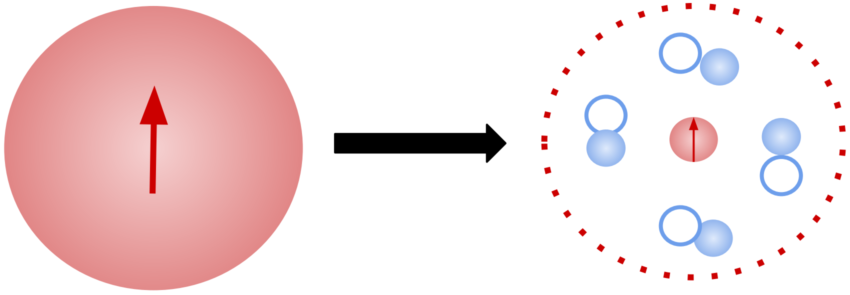

Pair of particle-hole excitations

Quasiparticle

How to derive Navier-Stokes equations ?

We close the equation by writing and inverting the equation for the dissipative term

$$\Pi = -\frac{4}{3}\eta\frac{\partial v_{||}}{\partial z}$$

Inverting the collision integral naturally leads to viscosity $\boxed{\eta \propto \left\langle \frac{1}{I^{l=2}(\epsilon)} \right\rangle_\epsilon}$

This method is exact, we do not use uncontrolled approximations such as <u>relaxation-time approximations</u> or <u>variational methods</u>

Finally we derive the linearized Navier-Stokes equation for the longitudinal velocity :

$$\boxed{m\rho_0\frac{\partial v_{||}}{\partial t} + \frac{1}{\rho_0\chi}\frac{\partial \rho}{\partial z} - \rho_0 F_{\text{ext}} = \frac{4}{3}\eta\frac{\partial^2 v_{||}}{\partial z^2}}$$

Transport coefficients of a low-temperature Fermi gas

Transport coefficient	Perturbative value ($\bar{a} = k_\mathrm{F} a$, $\tau_\sigma = 1/8ma^2T^2$)
$\eta = \frac{p_\mathrm{F}^2}{5m^*} F_2^s \rho_0 \tau_\eta$	$0.5133 \, [1 + 0.2633 \, \bar{a} \, + \mathcal{O}(\bar{a}^2)] \, \frac{p_\mathrm{F}^2 \tau_\sigma}{2m} \rho_0$
$\kappa = \frac{p_\mathrm{F}^3}{9m^*} \tau_\kappa$	$0.0831 \, [1 - 0.0875 \, \bar{a} + \mathcal{O}(\bar{a}^2)] \, \frac{p_\mathrm{F}^3 \tau_\sigma}{m} T$
$D = \frac{p_\mathrm{F}^2}{3(m^*)^2} F_0^a F_1^a \tau_D$	$0.268 \, [1 - 0.7848 \, \bar{a} + \mathcal{O}(\bar{a}^2)] \, \frac{p_\mathrm{F}^2 \tau_\sigma}{m}$

Perturbative expression of the transport coefficient